Recap for a continuous rv $X$

**Density Function (pdf):** A function $f : \mathbb{R} \to [0, \infty)$ with $\int_{-\infty}^{\infty} f(x)\,dx = 1$ such that

$$P(X \in B) = \int_B f(x)\,dx$$

for all “measurable” $B$.

**Distribution Function (cdf):** For $a \in \mathbb{R}$, $F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$ and

$$f(a) = F'(a).$$

**Expectation:** We have $E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$ and $E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)\,dx$.

If $a$ and $b$ are constants, then

$$E(ax + b) = aE(X) + b$$

**Variance:**

$$\text{Var}(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2.$$ 

**Standard deviation:** $\sqrt{\text{Var}(X)}$
Uniform random variables

$X$ is a uniform random variable on $(a, b)$ if its density is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise}. \end{cases}$$

Notation: $X \sim U(a, b)$.

Note: $P(x_1 < X < x_2) = P(x_1 + h < X < x_2 + h)$ for $x_1, x_2, x_1 + h, x_2 + h \in (a, b)$.

Properties of Uniform rvs

If $X \sim U(a, b)$, then:

$E(X) =$

$Var(X) =$
Problem 12

A bus travels between the two cities A and B, which are 100 miles apart.
If the bus has a breakdown, the distance from the breakdown to city A has a U(0,100) distribution.
There is a bus service station in city A, in B, and in the center of the route between A and B.
It is suggested that it would be more efficient to have the three stations located 25, 50, and 75 miles, respectively, from A.
Do you agree? Why?

Questions?
Normal random variables

$X$ is a normal random variable with parameters $\mu$ and $\sigma^2$ if its density is

$$f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

Notation: $X \sim N(\mu, \sigma^2)$.

Importance of Normal rvs

**Note 1:** Importance: many real life distributions follow a normal density curve. We will see later in Section 8 that normal distributions arise in an important result known as Central Limit Theorem.
Normal rv contd.

**Note 2:** $\frac{1}{\sqrt{2\pi \sigma^2}}$ is to have $\int_{-\infty}^{\infty} f(x)dx = 1$.

Normal rv contd.

**Note 3:** If $X \sim N(\mu, \sigma^2)$ and $a, b$ are two numbers, then $aX + b \sim N(a\mu + b, a^2\sigma^2)$.

Terminology: $N(0, 1)$ is called a **standard normal random variable**. Note that, if $X \sim N(\mu, \sigma^2)$, then

$$\frac{X - \mu}{\sigma} \sim N(0, 1)$$

is a standard normal.

This procedure is called **standardizing**, and allows all computations involving $N(\mu, \sigma^2)$ to be reduced to those for $N(0, 1)$. 
Normal rv contd.

**Note 4:** $\mu = \mathbb{E}X$, $\sigma^2 = \text{Var}(X)$. 
Note 5: The cumulative distribution function of $N(0, 1)$ is

$$
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy, \quad -\infty < x < \infty,
$$

and cannot be computed explicitly.

For $x > 0$, $\Phi(x)$ are usually given in the textbook. For $x < 0$, use $\Phi(x) = 1 - \Phi(-x)$.
Example 4b

If $X$ is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find

(a) $P(2 < X < 5)$;

(b) $P(X > 0)$;

(c) $P(|X - 3| > 6)$.

Questions?
That's all Folks!