Recap

Independence and Disjointness are different.

Independent Trials.
Gambler’s ruin problem

- Two gamblers, Bob and Alice, bet on the outcomes of successive flips of a coin.
- On each flip, if the coin comes up heads, Bob collects 1 unit from Alice,
- whereas if it comes up tails, Bob pays 1 unit to Alice.
- They continue to do this until one of them runs out of money.
- Assume that the successive flips of the coin are independent and each flip results in a head with probability \( p \).

What is the probability that Bob ends up with all the money if he starts with \( i \) units and Alice starts with \( N - i \) units?

Solution

Let \( p_i = \) probability that Bob ends up with all the money where he starts with \( i \) units and Alice starts with \( N - i \) units.

Consider \( N \) as fixed.

Solving Gambler’s ruin problem

\[
p_i = p \cdot p_{i+1} + q \cdot p_{i-1}, \quad p_0 = \ , \quad p_N =
\]
Monty Hall Problem

- There are three doors, one door reveals a car and other two reveal goats.
- A contestant choose a door at random.
- Monty opens another door (at random) containing a goat and gives the option to switch to the unopened door.

Let $D_i$ be the event that the contestant choose door $i$ at the beginning, $C_i$ be the event that door $i$ has the car for $i = 1, 2, 3$ and $W$ be the event that the contestant wins by switching the choice. Then

\[
P(W \mid D_1) = \sum_{i=1}^{3} P(W \mid D_1, C_i)P(C_i \mid D_1) = \]
Monty Hall Problem

Thus, $P(\text{win by not switching} \mid D_1) =$

Questions?
Simpson’s Paradox

Consider two doctors Dr. Hibbert and Dr. Nick; Their success statistics for treating Heart diseases and flu infections¹:

<table>
<thead>
<tr>
<th></th>
<th>Heart</th>
<th>Flu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hibbert</td>
<td>S=70, F=20</td>
<td>S=10, F=00</td>
</tr>
<tr>
<td>Nick</td>
<td>S=01, F=09</td>
<td>S=81, F=09</td>
</tr>
</tbody>
</table>

Dr. Hibbert has success rate 77.77% in heart diseases and 100% in flu; Dr. Nick has success rate 10% in heart diseases and 90% in flu.

Combined

Dr. Hibbert has success rate 80%;
Dr. Nick has success rate 82%.

¹See also Berkeley Gender Bias case here

Explanation
Problem 3.60

The color of a person’s eyes is determined by a single pair of genes.

• If they are both blue-eyed genes, then the person will have blue eyes;
• if they are both brown-eyed genes, then the person will have brown eyes;
• if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes.

A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent.

Suppose that Smith and both of his parents have brown eyes, but Smith’s sister has blue eyes.

(a) What is the probability that Smith possesses a blue-eyed gene?

Problem 3.60

(b) Suppose that Smith’s wife has blue eyes. What is the probability that their first child will have blue eyes?

(c) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?
Questions?

That's all Folks!