HOMEWORK 8
Math 461: Probability Theory

Due date: March 30, 2023

Each problem is worth 10 points and only five randomly chosen problems will be graded. Please indicate whom you worked with, it will not affect your grade in any way.

(1) Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters \( \mu = 71 \) and \( \sigma^2 = 6.25 \). What percentage of 25-year-old men are over 6 feet, 2 inches tall? What percentage of men in the 6-footer club are over 6 feet, 5 inches?

(2) The width of a slot of a duralumin forging is (in inches) normally distributed with \( \mu = .900 \) and \( \sigma = .003 \).
(a) What percentage of forgings will be defective?
(b) What is the maximum allowable value of \( \sigma \) that will permit no more than 6 in 1000 defectives when the widths are normally distributed with \( \mu = .9000 \) and \( \sigma \)?

(3) One thousand independent rolls of a fair die will be made. Compute an approximation to the probability that the number 6 will appear between 150 and 200 times inclusively. If the number 6 appears exactly 200 times, find the probability that the number 5 will appear strictly less than 150 times.

(4) Twelve percent of the population is left handed. Approximate the probability that there are at least 20 left-handers in a school of 200 students. State your assumptions.

(5) Jones figures that the total number of thousands of miles that an auto can be driven before it would need to be junked is an exponential random variable with mean 20. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed, but rather is (in thousands of miles) uniformly distributed over (0, 40).

(6) If \( X \) is uniformly distributed over \((-1, 1)\), find (a) \( P(|X| > 1/2) \) and (b) the density function of the random variable \(|X|\).

(7) If \( X \) is an exponential random variable with parameter \( \lambda = 1 \), compute the probability density function of the random variable \( Y \) defined by \( Y = \log X \).

(8) Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let \( X_i \) equal 1 if the \( i \)-th ball selected is white, and let it equal 0 otherwise. Give the joint probability mass function of
(a) \( X_1, X_2 \);
(b) \( X_1, X_2, X_3 \).

(9) The joint probability density function of \( X \) and \( Y \) is given by
\[
f(x, y) = \frac{6}{7} \left( x^2 + \frac{xy}{2} \right), 0 < x < 1, 0 < y < 2
\]
and 0 otherwise.
(a) Verify that this is indeed a joint density function.
(b) Compute the density function of \( X \).
(c) Find \( P(X > Y) \).
(d) Find \( P(Y > 1 \mid X < 1/2) \).
(e) Find \( E(X) \).
(f) Find \( E(Y) \).

(10) A man and a woman agree to meet at a certain location about 12:30PM. If the man arrives at a time uniformly distributed between 12:15 and 12:45, and if the woman independently arrives at a time uniformly distributed between 12:00 and 1PM, find the probability that the first to arrive waits no longer than 5 minutes. What is the probability that the man arrives first?