Each problem is worth 10 points and only five randomly chosen problems will be graded. Please indicate whom you worked with; it will not affect your grade in any way.

(1) Let $X$ and $Y$ be positive random variables, not necessarily independent. Write the most appropriate of $\leq, \geq, =,$ or ? in the blank for each part (where “?” means that no relation holds in general.)

(a) $16 \cdot P(|X + Y| > 2) \quad \underline{\quad} \quad E((X + Y)^2)$
(b) $P(X + Y = 2) \quad \underline{\quad} \quad P(X = 1)P(Y = 1)$
(c) $E(X^2 + Y^2) \quad \underline{\quad} \quad E(2XY)$

(2) Suppose that $X$ is a random variable with mean and variance both equal to 20. What can be said about $P(0 < X < 40)$?

(3) From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.

(a) Give an upper bound for the probability that a student’s test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student’s test score is equal to 25.
(b) What can be said about the probability that a student will score between 65 and 85?
(c) How many students would have to take the examination to ensure, with probability at least .9, that the class average would be within 5 of 75? Do not use the central limit theorem.

(4) Fifty numbers are rounded off to the nearest integer and then summed. If the individual round-off errors are uniformly distributed over $(-.5,.5)$, approximate the probability that the resultant sum differs from the exact sum by more than 3.

(5) A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.