[2 × 5 pts.] Label the following statements as True or False. Briefly explain for partial credit.

(a) If \( V = \text{span}(S) \), then every vector in \( V \) can be written uniquely as a linear combination of vectors in \( S \).

\[ \text{Solution: False. We have } \mathbb{R}^2 = \text{span}((1, 0), (0, 1), (1, 1)) \text{ but } (1, 1) = (1, 0) + (0, 1). \]

(b) For two subsets \( S, T \subseteq V \), \( \text{span}(S) = \text{span}(T) \) implies that \( S = T \).

\[ \text{Solution: False. We have } \text{span}\{(1, 0)\} = \text{span}\{(1, 0), (2, 0)\}. \]

(c) If \( S \) is a linearly dependent set, then each vector in \( S \) is a linear combination of other vectors in \( S \).

\[ \text{Solution: False. Take } S = \{(0, 0), (1, 0)\}. \]

(d) The matrix \( A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) is in RREF.

\[ \text{Solution: False. The last column is a pivot column but a non-zero entry 2 other than the leading 1.} \]

(e) For the augmented matrix \( M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) the linear system \( LS(M) \) has infinitely many solutions.

\[ \text{Solution: False. No solution as the last column is a pivot column.} \]