Quiz 2
Math 416: Linear Algebra - Feb. 16, 2018
Please answer on this sheet.

Name: ____________________________ NetID: ________

[2 + 2 + 2 + 4 pts.] Answer the following questions.

(a) If \( V = \text{span}(S) \), then every vector in \( V \) can be written uniquely as a linear combination of vectors in \( S \). **True or False? Explain.**

**Solution:** False. We have \( \mathbb{R}^2 = \text{span}((1,0),(0,1),(1,1)) \) but \( (1,1) = (1,0) + (0,1) \).

(b) A finite dimensional vector space has finitely many bases. **True or False? Explain.**

**Solution:** False. Given a basis \( \beta = \{u_1, u_2, \ldots, u_n\} \), the set \( \{cu_1, cu_2, \ldots, cu_n\} \) is also a basis for all \( c \neq 0 \).

(c) The matrix \( A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) is in RREF. **True or False? Explain.**

**Solution:** False. The last column is a pivot column but a non-zero entry 2 other than the leading 1.

(d) Find the null space of the matrix \( A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix} \), a basis of it and its dimension.

**Solution:** \( A \) is row equivalent to \( \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \). Thus the null space of \( A \) is \( \{ (x_1, x_2, x_3) \mid x_1 + x_3 = 0, x_2 - x_3 = 0 \} = \{ (-t, t, t) \mid t \in \mathbb{R} \} \), a basis for the null space of \( A \) is \( \{ (-1, 1, 1) \} \) and dimension of the null space is 1.