Problem 1. A basis \((e_n)_n\) of a Banach space \(X\) is called bimonotone if for every \(\ell \leq m \leq n \in \mathbb{N}\) and real numbers \(a_1, \ldots, a_n\) we have
\[
\left\| \sum_{i=\ell}^{m} a_i e_i \right\| \leq \left\| \sum_{i=1}^{n} a_i e_i \right\|
\]
Prove that Banach space with a Schauder basis admits an equivalent norm with respect to which the given basis is bimonotone.

Problem 2. Let \((e_n)_n\) denote the unit vector basis of \(c_0\) and define \(s_n = \sum_{i=1}^{n} e_i\) for each \(n \in \mathbb{N}\). We call \((s_n)_n\) the summing basis of \(c_0\).

(i) Show that \((s_n)_n\) is a Schauder basis of \(c_0\).

(ii) Find a permutation of \((s_n)_n\) that is not Schauder basic.

(iii) Given an infinite sequence of real numbers \((a_n)_n\), show that \(\sum_{n=1}^{\infty} a_n s_n\) converges (i.e., there exists \(x \in c_0\) with \(\lim_{n} \| x - \sum_{i=1}^{n} a_i s_i \| = 0\)) if and only if \(\sum_{n=1}^{\infty} a_n\) converges (to some \(a \in \mathbb{R}\)).

Problem 3. Let \(1 \leq p < \infty\) and let \((e_n)_n\) denote the unit vector basis of \(\ell^p\). Define \(d_1 = e_1\) and \(d_n = e_n - e_{n-1}\) for \(n \geq 2\).

(i) Show that \((d_n)_n\) is linearly independent and \(\langle \{d_n : n \in \mathbb{N} \} \rangle = \ell^p\).

(ii) If \(p = 1\), show that \((d_n)_n\) is a Schauder basis of \(\ell^1\).

(iii) If \(p = 1\), find a permutation of \((d_n)_n\) that is not Schauder basic.

(iv) If \(1 < p < \infty\), show that no permutation of \((d_n)_n\) is Schauder basic.