Let $\mathbf{u}$ be a vector in $\mathbb{R}^3$. 
Let $u$ be a vector in $\mathbb{R}^3$.

- A linear combination of $u$ is any vector $tu$. 

- The span of $u$ is the set of all vectors $tu$: $\text{span}\{u\} = \{tu \mid t \in \mathbb{R}\}$

- A line parallel to $u$ which goes through the origin $\text{span}\{u\}$ is a subspace.
Let $\mathbf{u}$ be a vector in $\mathbb{R}^3$.

- A **linear combination** of $\mathbf{u}$ is any vector $t\mathbf{u}$.
- Any vector parallel to $\mathbf{u}$.
Let $u$ be a vector in $\mathbb{R}^3$.

- a **linear combination** of $u$ is any vector $tu$
  - any vector parallel to $u$.
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Let \( \mathbf{u} \) be a vector in \( \mathbb{R}^3 \).

- a **linear combination** of \( \mathbf{u} \) is any vector \( t \mathbf{u} \)
  - any vector parallel to \( \mathbf{u} \).
- the **span** of \( \mathbf{u} \) is the set of ALL vectors \( t \mathbf{u} \): \( \text{span}\{\mathbf{u}\} = \{ t \mathbf{u} \mid t \in \mathbb{R} \} \)
  - a line parallel to \( \mathbf{u} \) which goes through the origin
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  - a line parallel to $\mathbf{u}$ which goes through the origin
- $\text{span}\{\mathbf{u}\}$ is a **subspace**.
The span of the vector $\mathbf{u}$ is the line $\overrightarrow{OP} = t\mathbf{u}$.
Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be two vectors in $\mathbb{R}^3$. 

A linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$ is any vector $s \mathbf{u}_1 + t \mathbf{u}_2$. 

The span of $\mathbf{u}_1$ and $\mathbf{u}_2$ is the set of all vectors $s \mathbf{u}_1 + t \mathbf{u}_2$. 

If $\mathbf{u}_1$ and $\mathbf{u}_2$ are parallel, then the span is a line parallel to $\mathbf{u}_1$ and $\mathbf{u}_2$ that passes through the origin. 

If $\mathbf{u}_1$ and $\mathbf{u}_2$ are not parallel, then the span is a plane that contains $\mathbf{u}_1$ and $\mathbf{u}_2$ and passes through the origin. 

The span $\{\mathbf{u}_1, \mathbf{u}_2\}$ is a subspace.
Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be two vectors in $\mathbb{R}^3$.

- A **linear combination** of $\mathbf{u}_1$ and $\mathbf{u}_2$ is any vector $s\mathbf{u}_1 + t\mathbf{u}_2$.

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1. If \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) are parallel, then the span is a line parallel to \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) that passes through the origin.

2. If \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) are NOT parallel, then the span is a plane that contains \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) and passes through the origin.
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- \( \text{span}\{\mathbf{u}_1, \mathbf{u}_2\} \) is a **subspace**.
The span of noncollinear vectors $u_1, u_2$
Let $u_1$, $u_2$, and $u_3$ be three vectors in $\mathbb{R}^3$. 

...
Let $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ be three vectors in $\mathbb{R}^3$.

- A **linear combination** of $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ is any vector $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$. 

The span of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ is the set of all vectors $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$.

1. If all $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are parallel, then the span is a line that passes through the origin.

2. If $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are not all parallel but they are in the same plane, then the span is that plane (which passes through the origin).

3. If $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ do not lie in the same plane, then the span is $\mathbb{R}^3$. 

The span of the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a **subspace**.
Let $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ be three vectors in $\mathbb{R}^3$.

- A **linear combination** of $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ is any vector $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$.

- The **span** of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ is the set of all vectors $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$.

- If all $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are parallel, then the span is a line that passes through the origin.

- If $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are not all parallel but they are in the same plane, then the span is that plane (which passes through the origin).

- If $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ do not lie in the same plane, then the span is $\mathbb{R}^3$.

The span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a subspace.
Let $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ be three vectors in $\mathbb{R}^3$.

- A **linear combination** of $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ is any vector $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$.

- The **span** of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ is the set of ALL vectors $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$.

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2. If $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are not all parallel but they are in the same plane, then the span is that plane (which passes through the origin).
Let $u_1$, $u_2$, and $u_3$ be three vectors in $\mathbb{R}^3$.

- A **linear combination** of $u_1$, $u_2$, $u_3$ is any vector $au_1 + bu_2 + cu_3$.

- The **span** of $u_1$, $u_2$, and $u_3$ is the set of all vectors $au_1 + bu_2 + cu_3$.

1. If all $u_1$, $u_2$, and $u_3$ are parallel, then the span is a line that passes through the origin.

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3. If $u_1$, $u_2$, and $u_3$ do not lie in the same plane, then the span is $\mathbb{R}^3$. 

\[ \text{span} \{ u_1, u_2, u_3 \} \text{ is a subspace.} \]
Let $u_1$, $u_2$, and $u_3$ be three vectors in $\mathbb{R}^3$.

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- $\text{span}\{u_1, u_2, u_3\}$ is a **subspace**.
The span of \( \{u_1, u_2, u_3\} \) is \( \mathbb{R}^3 \)
To determine whether vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots \mathbf{u}_k$ in $\mathbb{R}^3$ span a line, or a plane, or all of $\mathbb{R}^3$, find the RREF of the matrix $A = [\mathbf{u}_1 \mathbf{u}_2 \ldots \mathbf{u}_k]$.

- RREF of $A$ has 0 non-zero rows $\Rightarrow$ the span is a point (the origin)
- RREF of $A$ has 1 non-zero row $\Rightarrow$ the span is a line
- RREF of $A$ has 2 non-zero rows $\Rightarrow$ the span is a plane
- RREF of $A$ has 3 non-zero rows $\Rightarrow$ the span is all of $\mathbb{R}^3$

Calculator tip: If your matrix has more rows than columns, add zero column(s) to create a square matrix. Otherwise your calculator will report an error.
**Definition**: A non-empty subset $V$ of vectors in $\mathbb{R}^n$ is a **subspace** of $\mathbb{R}^n$ if:

1. **Closure under vector addition**: Whenever $u$ and $v$ are in $V$, then $u + v$ is in $V$.
2. **Closure under multiplication by real numbers**: Whenever $u$ is in $V$, then $cu$ is in $V$ for any real number $c$. 
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- whenever $\mathbf{u}$ is in $V$, then $c\mathbf{u}$ is in $V$ for any real number $c$
  (closure under multiplication by real numbers)