Exam 3 Review
Fall 2017

Date: Monday, December 4, 7pm, arrive at 6:30pm
BRING: your student ID, calculator, pencil, eraser, regular pen

Rooms:
Section A1 (8am lecture): 314 Altgeld Hall (different than last time!)
Section F1 (2pm lecture): Gregory Hall 112
Section G1 (3pm lecture): Loomis Laboratory 141

Today: Answer at least 75% of iClicker questions to receive 5 iClicker points.

1. Let \( S = \{(2/3, -3, 1/2), (-4, 18, -3), (8/3, -12, 2)\} \).
   Give a geometric description of the span of \( S \).
   (a) All of \( \mathbb{R}^2 \)
   (b) A plane in \( \mathbb{R}^3 \)
   (c) A line in \( \mathbb{R}^3 \)
   (d) A point in \( \mathbb{R}^3 \)
   (e) All of \( \mathbb{R}^3 \)

2. What is the null space of the following matrix \( A \)?

\[
A = \begin{bmatrix}
7 & -5 & 3 \\
2 & 1 & -3 \\
4 & -2 & 6
\end{bmatrix}
\]

(A) \( \text{span}\{(7, -5, 3)\} \)
(B) \( \{(0, 0, 0)\} \)
(C) All of \( \mathbb{R}^3 \).
(D) \( \text{span}\{(2, 5, 1)\} \)
(E) \( \text{span}\{(3, -2, 4), (-1, 1, -3)\} \)
3. What is the null space of the following matrix $A$?

$$A = \begin{bmatrix} 7 & -5 & 3 \\ 2 & 1 & -3 \\ 5 & -6 & 6 \end{bmatrix}$$

(A) span\{$(7, -5, 3)$\}
(B) $\{(0, 0, 0)\}$
(C) All of $\mathbb{R}^3$.
(D) span\{$(12, 27, 17)$\}
(E) span\{$(12, 27, 17), (7, -5, 3)$\}

4. The transition matrix of a certain Markov process is

$$T = \begin{bmatrix} .55 & .2 \\ .45 & .8 \end{bmatrix}$$

Find the exact stable vector of the process among the list:

(A) The correct answer is not here.
(B) $\begin{bmatrix} 7/13 \\ 6/13 \end{bmatrix}$
(C) $\begin{bmatrix} 6/13 \\ 7/13 \end{bmatrix}$
(D) $\begin{bmatrix} 5/13 \\ 8/13 \end{bmatrix}$
(E) $\begin{bmatrix} 4/13 \\ 9/13 \end{bmatrix}$

5. Consider below the transition matrix $T$. How many years will it take to stabilize at the stable distribution if we work to four decimal places?

$$T = \begin{bmatrix} .95 & .25 \\ .05 & .75 \end{bmatrix}$$

(A) 17 years.
(B) 20 years.
(C) 23 years.
(D) 26 years.
(E) 29 years.
6. Let $S$ be the set of vectors $\mathbf{u} = (-1, -3, 1)$, $\mathbf{v} = (1, 0, 1)$, and $\mathbf{w} = (-1, -6, 3)$. Which of the following claims is FALSE?

(A) The span of $S$ is a plane passing through the origin.
(B) The matrix whose columns are the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ is invertible.
(C) The span of $S$ is a subspace of $\mathbb{R}^3$.
(D) The set $S = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent.
(E) The point $(1, 9, -5)$ can be expressed as a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

7. Let

$$A = \begin{bmatrix} 5 & 1 & 7 \\ -8 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

Out of the following vectors:

$\vec{u} = (7, -2, -3)$
$\vec{v} = (4, -13, -5)$
$\vec{w} = (-5, 1, 8)$

which are in the column space of $A$?

(A) $\vec{w}$ only
(B) All of these vectors are in the column space of $A$
(C) $\vec{u}$ and $\vec{v}$
(D) $\vec{v}$ and $\vec{w}$
(E) None of these vectors are in the column space of $A$

8. Which of the following is a line in $\mathbb{R}^3$?

(A) the set of points $(x, y, z)$ such that $3x - 5y = 4$
(B) $\text{span}\{(1, -2, 1), (2, 6, -3)\}$
(C) the set of solutions $(x, y, z)$ of the system of equations $x + 2y - 5z = 3$ and $-x + y + 3z = 1$
(D) $\{(3t - s, t - s, 2t) \mid t, s \in \mathbb{R}\}$
(E) the set of points $(x, y, z) = (1, 1, 2) + \text{span}\{(1, -2, 1), (5, -3, 7)\}
9. Three of the following four statements are all equivalent to each other. Choose the one statement that is not equivalent to the other three.

(A) $V$ is a span of vectors in $\mathbb{R}^3$.
(B) $V$ is a subspace of $\mathbb{R}^3$.
(C) $V$ is linearly independent.
(D) $V$ is a subset of $\mathbb{R}^3$ that is closed under addition and scalar multiplication.

10. Consider the following statements:

(I) The span of three vectors in $\mathbb{R}^3$ can be a plane that passes through the origin.

(II) If $\mathbf{u}$ and $\mathbf{v}$ are two linearly dependent vectors in $\mathbb{R}^n$, then $\text{span}\{\mathbf{u}, \mathbf{v}\} = \text{span}\{\mathbf{v}\}$.

(III) The span of two vectors in $\mathbb{R}^2$ is always all of $\mathbb{R}^2$.

Which of the above statements are TRUE?

(A) Only (I) is true.
(B) Only (II) is true.
(C) Only (I) and (II) are true.
(D) Only (I) and (III) are true.
(E) All (I), (II), and (III) are true.
Solutions

1. C
2. B
3. D
4. E
5. D
6. B
7. B
8. C
9. C
10. C