Section 5.5: Bases and Basis Selection

Example 1: (Motivational Example)

1. Show that $\vec{v}_1 = (1, 3)$ and $\vec{v}_2 = (1, 1)$ are linearly independent.

2. Geometrically, what is $\text{span}\{\vec{v}_1, \vec{v}_2\}$?
Example 1 (Continued):

1. Let \( \vec{v}_1 = (1, 3), \vec{v}_2 = (1, 1), \) and \( \vec{v}_3 = (2, 4). \) What is \( \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}? \)

2. Let \( \vec{v}_1 = (1, 1) \) and \( \vec{v}_2 = (-3, -3). \) Geometrically, what is \( \text{span}\{\vec{v}_1, \vec{v}_2\}? \)
Definition: A finite set of vectors \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\} is a basis for a vector space \(V\) if

1. 

2. 

Definition: the standard basis for \(\mathbb{R}^n\) is the set of vectors:

\[
\begin{align*}
\vec{e}_1 &= (1, 0, 0, \ldots, 0, 0) \\
\vec{e}_2 &= (0, 1, 0, \ldots, 0, 0) \\
\vec{e}_3 &= (0, 0, 1, \ldots, 0, 0) \\
& \vdots \quad \vdots \\
\vec{e}_n &= (0, 0, 0, \ldots, 0, 1)
\end{align*}
\]

Example 2: Describe the vector \((\pi, -7, 5, \frac{1}{2})\) as linear combinations of the standard basis vectors.
Example 3: (Selecting a basis from a spanning set.) Let \( \vec{v}_1 = (1, -2), \ \vec{v}_2 = (-3, 6), \) and \( \vec{v}_3 = (1, 1). \) Find a basis for the subspace formed by \( \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}. \)
Example 4: (Selecting a basis from a spanning set.) Let \( \vec{v}_1 = (-1, -4, 3) \), \( \vec{v}_2 = (3, 11, -5) \), \( \vec{v}_3 = (3, 10, -1) \), \( \vec{v}_4 = (2, 7, 0) \), and \( \vec{v}_5 = (-1, -3, -7) \). Find a basis for the subspace \( W \) formed by \( \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\} \).
Remarks:

**Theorem:** Let $W$ be a subspace of $\mathbb{R}^n$. Then every basis for $W$ has .................

**Definition:** *the dimension of a vector space*
Remark:

This definition agrees with the geometric concept of dimension:

<table>
<thead>
<tr>
<th>basis</th>
<th># of vectors in a basis (= dimension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{\vec{v}_1}</td>
<td>1, a line (looks like (\mathbb{R}))</td>
</tr>
<tr>
<td>{\vec{v}_1, \vec{v}_2}</td>
<td>2, a plane (looks like (\mathbb{R}^2))</td>
</tr>
<tr>
<td>{\vec{v}_1, \vec{v}_2, \vec{v}_3}</td>
<td>3, a 3-D space (looks like (\mathbb{R}^3))</td>
</tr>
</tbody>
</table>

Example 5: The dimension of the subspace from Example 3
Example 6: Consider the following information about cottage cheese, cultured milk, Swiss cheese, and yogurt.

<table>
<thead>
<tr>
<th></th>
<th>Per ounce of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cottage cheese</td>
</tr>
<tr>
<td>calories</td>
<td>25</td>
</tr>
<tr>
<td>protein (g)</td>
<td>4</td>
</tr>
<tr>
<td>lysine (mg)</td>
<td>300</td>
</tr>
<tr>
<td>cost (cents)</td>
<td>7</td>
</tr>
</tbody>
</table>

Use a dependency table to find the makeup of synthetic Swiss cheese using the other three basic foods and compare its cost to real Swiss cheese.
Example 6 (Continued):