Section 5.3: Subspaces

Definition: A non-empty subset $V$ of vectors in $\mathbb{R}^n$ is a **subspace** of $\mathbb{R}^n$ if:

Proposition: Every subspace must contain ________________.

Example 1: Show that the line $(-3t, t, 8t)$ for all $t \in \mathbb{R}$ is a subspace of $\mathbb{R}^3$. 
Example 2: Show that the line \((-3t + 2, t, 8t)\) for all \(t \in \mathbb{R}\) is NOT a subspace of \(\mathbb{R}^3\).

Example 3: Show that the plane defined by \((s + 3t, 5s + 6t, 2s + 2t)\) for all \(s, t \in \mathbb{R}\) is a subspace of \(\mathbb{R}^3\).
Theorem:

Subspaces Associated with a Matrix

Definition: *null space*

**Example 4:** Find the null space of

$$A = \begin{bmatrix} 3 & 6 & 12 \\ -3 & -4 & -10 \end{bmatrix}$$

and write the null space as a span.
Definition: *column space*

**Example 5:** Determine whether $\vec{u} = (-3, 6, 6)$ and $\vec{v} = (6, 6, -3)$ are in the column space of

$$A = \begin{bmatrix}
  3 & 6 & -3 \\
  3 & 3 & -3 \\
-6 & -12 & 6
\end{bmatrix}$$

And write each one as a linear combination of the column vectors, if possible.
Example 6: Solve the following linear system:

\[
\begin{align*}
    x_1 + x_2 + 2x_3 &= 8 \\
    -x_1 - 2x_2 + 3x_3 &= 1 \\
    3x_1 - 7x_2 + 4x_3 &= 10
\end{align*}
\]

The Consistency Theorem (Version 3):
Example 7: A paper company wants to make 2 new kinds of paper. Paper 1 is made from 60% white pine, 32% black pine, and 8% scotch pine. Paper 2 is made from 73% white pine, 20% black pine, and 7% scotch pine. The trees they have available are in two mixes on two equal sized tracts of land. The first forest contains 84% white pine, 12% black, and 4% scotch pine. The second forest contains 40% white pine, 44% black, and 16% scotch pine. It is not financially prudent to sort the wood. Is it possible to make either of these papers with the current mixes available?