Section 3.3: Linear Combinations

Definition: *linear combination*

**Example 1:** In \( \mathbb{R}^3 \), let \( \vec{u}_1 = (1, 2, 1) \), \( \vec{u}_2 = (1, 0, 2) \), and \( \vec{u}_3 = (1, 1, 0) \). Determine whether the vector \( \vec{v} = (2, 1, 5) \) can be written as a linear combination of \( \vec{u}_1 \), \( \vec{u}_2 \), and \( \vec{u}_3 \).
Example 1 (Continued):
Definition: a vector (as a matrix)

A vector is a matrix consisting of a single column: \( \vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \)

Example 2: Comparing the Old and the New:

\[
\begin{align*}
x + 5y &= 7 \\
2x + 3y &= 3 \\
3x + 2y &= 7
\end{align*}
\]
Example 3: Rewrite the following linear system as a vector equation and then solve the equation.

\[
\begin{align*}
x + 2y + 3z &= 4 \\
4x + 5y + 6z &= 10 \\
7x + 8y + 9z &= 16
\end{align*}
\]
The Consistency Theorem (Version 2):

Example 4: Solve the following system as a linear combination of the column vectors of $A$.

$$
\begin{bmatrix}
1 & 2 & 4 & 2 \\
3 & 5 & 7 & 9 \\
-1 & -3 & -9 & 1
\end{bmatrix}
$$
Example 4 (Continued):

Example 5: Solve the following system as a linear combination of the column vectors of $A$.

\[
\begin{bmatrix}
1 & 2 & 4 & 3 \\
3 & 5 & 7 & 5 \\
-1 & -3 & -9 & 2
\end{bmatrix}
\]
Example 6: A paper company produces 3 kinds of paper at 3 different mills. The Huntington mill produces 3 tons of basic white paper, 3 tons of color paper, and 6 tons of stationary paper per 8-hour work day. The Charleston mill produces 6 tons of basic white paper, 8 tons of color paper, and 10 tons of stationary paper per 8-hour work day. The Parkersburg mill produces 9 tons of basic white paper, 10 tons of color paper, and 17 tons of stationary paper per 8-hour work day. The Parkersburg mill is expensive to run, so the company is exploring shutting it down. Is it possible that the other two mills can be run overtime to exactly make up for the daily output lost by shutting down the Parkersburg mill?