Section 2.5: Linear Programming and the Simplex Algorithm

**Definition:** *The Simplex Algorithm*

**Requirements for Using the Simplex Algorithm:**

1. The linear program must be a maximization.
2. The constraint inequalities must have the form:
   
   \[
   \text{linear expression} \leq \text{a positive number}
   \]
3. All variables must be non-negative.

**Definition:** *slack variable*
Definition: initial simplex table

Example 1: Find the initial simplex table for the following linear program:

\[
\begin{align*}
\text{maximize} & \quad z = 8x_1 + 6x_2 \\
\text{subject to} & \quad 2x_1 + 4x_2 \leq 6 \\
& \quad x_1 + 7x_2 \leq 4 \\
& \quad x_1 \geq 0 \\
& \quad x_2 \geq 0
\end{align*}
\]
The Simplex Algorithm:

1. Choose the **pivot column**, the column with the most negative entry in the top row.

2. All positive entries in the pivot column are candidates for pivoting. For each positive entry, divide it into the positive entry in the solution or constant column. Define the pivot point to be the positive entry in the pivot column that corresponds to the smallest quotient.

3. Pivot on the entry chosen in Step 2.

4. Repeat Steps 1 to 3 until all the entries in the top row are positive.

5. The maximum $z$ value is the number at the top of the solution or constant column.

**Example 2:** Use the simplex algorithm to maximize the $z$ of Example 1 over the same feasibility region.
Procedure for defining the basic feasible solution (where $x$ attains its maximum value) of a given simplex table:

Example 3: Find the basic feasible solution for the final simplex table of Example 2 and solve the linear program from Example 1.
Example 4: Use the simplex algorithm to solve the following linear program:

maximize \( z = 2x_1 + 3x_2 \)

subject to
\[
\begin{align*}
4x_1 + 2x_2 &\leq 9 \\
x_1 + 3x_2 &\leq 7 \\
x_1 &\geq 0 \\
x_2 &\geq 0
\end{align*}
\]
**Example 5:** A toy company makes Dracula, Frankenstein, and Wolfman masks. A Dracula mask requires 8 ounces of hair, 8 ounces of plastic, and 4 ounces of latex, and sells for 60 dollars. A Frankenstein mask requires 4 ounces of hair, 1 ounce of plastic, 4 ounces of latex, and sells for 70 dollars. A Wolfman mask requires 10 ounces of hair, 2 ounces of plastic, 4 ounces of latex, and sells for 80 dollars. The company possesses 500 ounces of hair, 40 ounces of plastic, and 140 ounces of latex. Assuming that the company will sell all of the masks produced, maximize the revenue given the company’s resources.
Example 5 (Continued):