Section 2.2: Reduced Row Echelon Form

Definition: reduced row echelon form

Definition: back addition

A Procedure for Moving a Matrix into Reduced Row Echelon Form:
Example 1: (Gaussian elimination with back addition.) Put the following matrix into reduced row echelon form.

\[
\begin{bmatrix}
1 & 2 & 1 & 4 \\
3 & 7 & -2 & 1 \\
-2 & 3 & 3 & -1
\end{bmatrix}
\]
Calculator Commands:

1. entering a matrix (supplemental section S-1 and 2)

2. to do row operations (S-6 and 7)

3. computing row echelon form

4. computing reduced row echelon form
Example 2: Use Gaussian elimination and back addition to solve the following system of equations.

\[ x_1 + 2x_2 + 3x_3 = 4 \]
\[ 5x_1 + 6x_2 + 7x_3 = 8 \]
\[ x_1 + 2x_2 + x_3 = 2 \]
Definition: *multisystem*

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**Example 3:** Solve the given systems of equations simultaneously.

<table>
<thead>
<tr>
<th>System A</th>
<th>and</th>
<th>System B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 2y - 3z = -1$</td>
<td>$x + 2y - 3z = 2$</td>
<td></td>
</tr>
<tr>
<td>$3x + 5y + 3z = 2$</td>
<td>$3x + 5y + 3z = -3$</td>
<td></td>
</tr>
<tr>
<td>$-5x - 9y + 8z = 5$</td>
<td>$-5x - 9y + 8z = -16$</td>
<td></td>
</tr>
</tbody>
</table>
Example 4: The Super Ball Company makes volleyballs, basketballs, and waterpolo balls. Each volleyball uses 4 ounces of latex, 1 ounce of nylon, and 2 ounces of vinyl in its construction. Each basketball is made of 7 ounces of latex, 2 ounce of nylon, and 5 ounces of vinyl. Each waterpolo ball is made of 2 ounces of latex, 1 ounce of nylon, and 5 ounces of vinyl. The company has two factories. Factory A has 325 ounces of latex, 100 ounces of nylon, and 300 ounces of vinyl on hand. Factory B has 400 ounces of latex, 150 ounces of nylon, and 600 ounces of vinyl on hand. Find the amount of each type of ball that each factory should produce if the company’s desire is to use all of the resources it has on hand.