Section 1.2: Solving Linear Programs in the Plane

Definition: a linear function

Example 1: Consider the feasibility region for the shingle company from Lesson 1.3, Example 3. Now suppose each bundle of shingles sells for $10. Then the revenue $R$ can be written as a linear function of $x$ and $y$. Specifically, $R = 10x + 10y$ where $x$ is the number of the first kind of shingles sold and $y$ is the number of the second kind of shingles sold. (Note that different values of $R$ yield different lines with the exact same slope.)

Choosing different values of $R$, graph some of the lines for $R = 10x + 10y$ over the feasibility region from the shingle company example. Can you draw any conclusions about what type of shingles to produce?
Definition: *lines of constancy*

Definition: *a linear program*

**Example 2:** Minimize $z = -2x + y$ subject to the following constraints:

\[
\begin{align*}
x - y & \geq -3 \\
x + y & \leq 5 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]
Example 2 (Continued):

Example 3: Find the maximum and minimum values of the linear function $z = -x + 3y$ over the rectangular region spanned by the points $(-1, 1), (-1, -2), (3, -2), (3, 1)$.
**Definition:** *polygon*

**Theorem:** Let $z = ax + by$ be a linear function, and let $P$ be a polygon in the plane. Then the maximum and minimum values of $z$ are attained at corner points of $P$. 
Example 4: Minimize $z = -2x + y$ subject to the following constraints:

\[
\begin{align*}
    x - y & \geq -3 \\
    x + y & \leq 5 \\
    x & \geq 0 \\
    y & \geq 0
\end{align*}
\]
Example 5: Maximize $z = x - 3y$ subject to the following constraints:

\[
\begin{align*}
7x + 2y & \leq 14 \\
-3x + y & \leq 3 \\
y & \geq 0
\end{align*}
\]