Math 125 Practice Exam 2  
Monday, October 24th, 2016  

Name: _______________________________ UIN: _________________________

Circle the section you will pick up your exam in:

   8am       2pm       3pm

A. **CODE THE LETTERS BELOW ON YOUR SCANTRON NOW**

B. Multiple Choice questions begin at number 3 on the scantron.

C. If you have a question, raise your hand and a proctor will come to you. If you have to use the bathroom, do so NOW. You will not be permitted to leave the room and return during the exam.

D. No cells phones, i-Pods, MP3 players. Turn them off now. If you are seen these items in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece.

E. If you finish early, quietly and respectfully get up and hand in your exam.

F. When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

G. To ensure that you receive full credit, show all of your work.

H. Good luck. You have 60 minutes to complete this exam.
1. (20 points) The Fragile Glass bottle company produces glass bottles from recycled white, brown and green glass at 3 different factories. Listed below are the number of batches of each color bottle the factories produce in a daily 8 hour shift.

<table>
<thead>
<tr>
<th></th>
<th>Peoria Factory</th>
<th>Urbana Factory</th>
<th>Bloomington Factory</th>
</tr>
</thead>
<tbody>
<tr>
<td>white glass</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>brown glass</td>
<td>3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>green glass</td>
<td>1</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

(A) (10 points) Is it possible to close the Bloomington Factory, and replace it’s production EXACTLY by running the Peoria and Urbana Factories overtime?

(B) (10 points) They are offered a weekly contract for 21 batches of white glass bottles, 31 batches of brown glass bottles, and 37 batches of green glass bottles. Can they fill this contract? If so, give a valid production schedule.
2. (20 points) An economy has 3 different sectors: construction, transportation, and tourism. Producing $1 worth of construction takes $0.37 of construction, $0.26 of transportation, and $0.27 of tourism. Producing $1 of tourism takes $0.19 of construction, $0.20 of transportation, and $0.20 of tourism. Producing $1 of tourism takes $0.23 of construction, $0.26 of transportation, and $0.25 of tourism.

(a) Fill in the consumption matrix for this economy below.

\[
C = \begin{bmatrix}
\text{Const} & \text{Trans} & \text{Tour} \\
\text{Construction} \\
\text{Transportation} \\
\text{Tourism}
\end{bmatrix}
\]

(b) The society has a demand for $1650 in construction, $1700 in transportation, and $1190 in tourism. How much construction needs to be produced to meet this demand? Please show all work to receive full credit, and write your final answer in the box.

The economy should produce $\underline{1650}$ in construction to meet the total societal demand stated above.
3. (5 points) What is the length of the vector that starts at \((9, 4, -5)\) and ends at \((9, 7, -1)\)?

(A) \(\sqrt{45}\)

(B) The correct answer is not here.

(C) 4

(D) \(\sqrt{24}\)

(E) 5

4. (5 points) For what values of \(m\) and \(n\) are the vectors \((2, m, 5)\) and \((6, 12, n)\) parallel?

(A) \(m = 4, n = 15\)

(B) \(m = 12, n = 15\)

(C) The vectors are never parallel.

(D) \(m = 3, n = 15\)

(E) \(m = 4, n = 5\)

5. (5 points) Let \(A\) and \(B\) be \(4 \times 4\) matrices. Consider the statements:

(I) \(AB = BA\)

(II) \((A + B)^\top = A^\top + B^\top\)

(III) \((AB)^\top = A^\top B^\top\)

Which of the above statements are true for any \(A\) and \(B\)?

(A) Only (I) and (III) are true.

(B) Only (II) and (III) are true.

(C) Only (II) is true.

(D) (I), (II), and (III) are all true.

(E) None of these are true.
6. (5 points) The graphs below (drawn on the same scales) show two given vectors \( \vec{u} \) and \( \vec{v} \) and four other vectors. Which of \( \vec{p} \), \( \vec{q} \), \( \vec{r} \) and \( \vec{s} \), if any, is the vector \( \vec{u} + 2\vec{v} \)?

(A) \( \vec{r} \)

(B) None of these.

(C) \( \vec{p} \)

(D) \( \vec{q} \)

(E) \( \vec{s} \)
8. (5 points) The Halloween Mask Company makes two different types of masks from a supply of four raw materials: hair, plastic, latex and adhesive, and wishes to maximize its profit. This leads to the following initial table for the simplex algorithm:

\[
\begin{bmatrix}
1 & -70 & -60 & 0 & 0 & 0 & 0 \\
0 & 8 & 4 & 1 & 0 & 0 & 500 \\
0 & 8 & 1 & 0 & 1 & 0 & 250 \\
0 & 4 & 2 & 0 & 0 & 1 & 140 \\
0 & 1 & 1 & 0 & 0 & 1 & 50 \\
\end{bmatrix}
\]

oz hair

If the final simplex table for this problem is

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 7 & 0 & 12 & 1510 \\
0 & 0 & 0 & 1 & 6 & 0 & -8 & 33 & oz hair \\
0 & 0 & 1 & 0 & 6 & 0 & 8 & 10 & oz plastic \\
0 & 0 & 0 & 0 & -4 & 1 & 6 & 21 & oz latex \\
0 & 1 & 0 & 0 & 8 & 0 & 6 & 13 & oz adhesive \\
\end{bmatrix}
\]

then

(A) All the raw materials are all used up.

(B) None of these is correct.

(C) 33 oz of hair and 21 oz of latex are unused and the rest of the raw materials are all used up.

(D) 33 oz of hair are unused and the rest of the raw materials are all used up.

(E) 21 oz of latex are unused and the rest of the raw materials are all used up.

9. (5 points) Which of the following is a vector equation of the line through the points \((3,3,4,5)\) and \((-1,-2,1,3)\)?

(A) None of these is an equation of the line.

(B) \(\vec{x} = (3,3,4,5) + t(4,5,3,2)\)

(C) \(\vec{x} = (-1,-2,1,3) + t(3,3,4,5)\)

(D) \(\vec{x} = (3,3,4,5) + t(-1,-2,1,3)\)

(E) \(\vec{x} = (-4,-5,-3,-2) + t(3,3,4,5)\)
10. (5 points) Think of the system of equations

\[
\begin{align*}
3x_1 + 2x_2 + x_3 &= b_1 \\
2x_1 + x_2 + x_3 &= b_2 \\
4x_1 - 7x_2 + x_3 &= b_3
\end{align*}
\]

as a linear combination

\[
x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 = \vec{b}
\]

where \( \vec{a}_1, \vec{a}_2, \vec{a}_3 \) and \( \vec{b} \) are the column vectors:

\[
\vec{a}_1 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 2 \\ 1 \\ -7 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}
\]

Suppose we know that \( \vec{b} = 2\vec{a}_1 - 7\vec{a}_3 + \vec{a}_2 \). Which of the following is a solution of the linear system?

(A) \( \vec{x} = (2, -1, 3) \)

(B) \( \vec{x} = (-1, 2, 3) \)

(C) \( \vec{x} = (3, 2, -1) \)

(D) \( \vec{x} = (2, 1, -7) \)

(E) The correct answer is not here.
11. (5 points) Recall that every simplex table corresponds to a corner of the feasibility region of a maximization problem, and a pivot according to the simplex algorithm moves from one corner to another corner. If you follow the rules of the Simplex Algorithm to make a pivot on this table:

\[
\begin{bmatrix}
1 & -20 & 0 & -10 & 0 & 5 & 0 & 18 \\
0 & 5 & 0 & 4 & 1 & 3 & 0 & 15 \\
0 & 2 & 1 & 3 & 0 & 3 & 0 & 10 \\
0 & 3 & 0 & 3 & 0 & 2 & 1 & 10 \\
\end{bmatrix}
\]

you would be moving from what corner point to what other corner point?

(A) corner (0, 10, 0) to corner (5, 0, 0)

(B) corner (3, 4, 0) to corner (0, 10, 0)

(C) corner (0, 10, 0) to corner (10/3, 10/3, 0)

(D) corner (0, 10, 0) to corner (3, 4, 0)
12. (5 points) The graph below shows four vectors: \( \vec{p}, \vec{q}, \vec{r} \) and \( \vec{s} \).

![Diagram of vectors](image)

Of these four vectors, for which two vectors is the absolute value of the dot product smallest?

(A) \( \vec{p} \cdot \vec{q} \) is the smallest in absolute value.

(B) \( \vec{p} \cdot \vec{r} \) is the smallest in absolute value.

(C) \( \vec{p} \cdot \vec{s} \) is the smallest in absolute value.

(D) \( \vec{q} \cdot \vec{s} \) is the smallest in absolute value.

(E) \( \vec{s} \cdot \vec{r} \) is the smallest in absolute value.

13. (5 points) Find the dot product of the following 2 vectors:

\[ \vec{u} = (5, -1, 3), \quad \vec{v} = (4, -2, -2) \]

(A) 14

(B) 4

(C) 16

(D) (20, 2, -6)

(E) (9, -3, 1)
14. (5 points) For what value of $k$ are the vectors $(-2, k, 3)$ and $(20, 7, k)$ orthogonal (i.e., perpendicular)?

(A) -3
(B) 4
(C) 2
(D) -4
(E) 3