Math 125 Fall 2017 Exam 2  
Monday, October 30, 2017

Name: ___________________________  NetID: ________________________@illinois.edu

Signature: ___________________________

Circle your lecture:

8am lecture  2pm lecture  3pm lecture

• CODE THE LETTERS BELOW ON YOUR SCANTRON NOW

• Multiple choice questions begin at number 3 on the scantron.

• No cells phones, i-Pods, MP3 players. Turn them off now. If you are seen these items in hand during the exam it will be considered cheating and you will be asked to leave.

• There are 14 question in the exam, 2 of which are free response and 12 are multiple choice.

• In the free response part, you must show all your work to receive full credit.

• Showing your work is NOT required for the multiple choice part.

• You can use your calculator in every problem of this exam.

• No questions will be answered during the exam. If you are uncertain about something, put a note about it on your exam paper and then answer the question as best you can with the information that you are given.

• You have 60 minutes to complete this exam plus 5 minutes extra to fill in your scantron.

• If you finish early, quietly and respectfully get up and hand in your exam.

• When time is up, put down your writing utensil, close your exam, and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

89. B
90. A
91. A
92. A
93. A
94. E
95. D
96. A

1
Free-Response score:

1. (20 points) ________________

2. (20 points) ________________

Total (40 points): ___________
1. (20 points) In this problem, you will use the simplex algorithm to solve the linear program:

maximize: \[ R = -7x - 3y + 4z \]
subject to:
- \[ -4x + 2y + 2z \leq 2 \]
- \[ 5y + z \leq 10 \]
- \[ -3x + 2z \leq 3 \]
- \[ x \geq 0, y \geq 0, z \geq 0 \]

(You may use the Pivot program on your calculator, or perform pivoting on the scratch paper provided.)

A. (6 points) Construct the initial simplex table corresponding to the linear program above. Circle the first pivot point to be used in the simplex algorithm.

B. (4 points) Write down the intermediate simplex table you get after performing the first pivot. Circle the next pivot point to be used in the simplex algorithm.

C. (2 points) Write down the resulting final simplex table.

D. (3 points) What is the basic feasible solution of the final simplex table? _____________

E. (2 points) What is the maximum \( R \)-value in this linear program? _____________

F. (3 points) Recall that every simplex table corresponds to a corner of the feasibility region and the simplex algorithm moves along the edges of the feasibility region from one corner to another corner. Find the sequence of corner points for the simplex algorithm performed in parts A, B, and C.

−−−−−−→ −−−−−−→
Solution.

A. (6 points) Construct the initial simplex table corresponding to the linear program above. Circle the first pivot point to be used in the simplex algorithm.

\[
\begin{bmatrix}
1 & 7 & 3 & -4 & 0 & 0 & 0 & 0 \\
0 & -4 & 2 & 1 & 0 & 0 & 0 & 2 \\
0 & 0 & 5 & 1 & 0 & 1 & 0 & 10 \\
0 & -3 & 0 & 2 & 0 & 0 & 1 & 3 \\
\end{bmatrix}
\]

B. (4 points) Write down the intermediate simplex table you get after performing the first pivot. Circle the next pivot point to be used in the simplex algorithm.

\[
\begin{bmatrix}
1 & -1 & 7 & 0 & 2 & 0 & 0 & 4 \\
0 & -2 & 1 & 1 & 1/2 & 0 & 0 & 1 \\
0 & 2 & 4 & 0 & -1/2 & 1 & 0 & 9 \\
0 & -2 & 0 & -1 & 0 & 1 & 1 & \\
\end{bmatrix}
\]

C. (2 points) Write down the resulting final simplex table.

\[
\begin{bmatrix}
1 & 0 & 5 & 0 & 1 & 0 & 1 & 5 \\
0 & 0 & -3 & 1 & -3/2 & 0 & 2 & 3 \\
0 & 0 & 8 & 0 & 3/2 & 1 & -2 & 7 \\
0 & 1 & -2 & 0 & -1 & 0 & 1 & 1 \\
\end{bmatrix}
\]

D. (3 points) What is the basic feasible solution of the final simplex table? (1, 0, 3, 0, 7, 0)

E. (2 points) What is the maximum $R$-value in this linear program? $R_{\text{max}} = 5$

F. (3 points) Recall that every simplex table corresponds to a corner of the feasibility region and the simplex algorithm moves along the edges of the feasibility region from one corner to another corner. Find the sequence of corner points for the simplex algorithm performed in parts A, B, and C.

\[\begin{array}{c}
(0,0,0) \rightarrow (0,0,1) \rightarrow (1,0,3)
\end{array}\]
2. (20 points) The Illinois Power Company delivers products that serve two different economic sectors: Oil and Gas. The production of $1.00 worth of oil requires $0.7 of oil and $0.4 of gas. The production of $1.00 worth of gas requires $0.35 of oil and $0.2 of gas.

(a) (5 points) The consumption matrix for the part of the economy served by this company is:

\[
C = \begin{bmatrix}
\text{Oil} & \text{Gas} \\
\text{oil} & \text{gas}
\end{bmatrix}
\]

(b) (2 points) Interpretation of Column 1: To make $1.00 worth of oil, the company requires $\square$ in raw materials.

Interpretation of Row 2: In the production $1.00 of oil and $1.00 of gas, the company consumes $\square$ in gas.

(c) (2 points) Which sectors of the economy are profitable? Oil sector Gas sector

(d) (5 points) Find the matrices \(I - C\) and \((I - C)^{-1}\).

\[
I - C = \begin{bmatrix}
\text{Oil} & \text{Gas} \\
\text{oil} & \text{gas}
\end{bmatrix} \quad (I - C)^{-1} = \begin{bmatrix}
\text{Oil} & \text{Gas} \\
\text{oil} & \text{gas}
\end{bmatrix}
\]

Is the economy serviced by this company productive? YES NO

(e) (5 points) Societal demands for what the Illinois Power Company produces are $100 for oil and $200 for gas. Is there a production schedule that will meet this demand? If no, enter NO in each of the following spaces. If yes, then the Illinois Power Company needs to produce

$\square$ in oil,

$\square$ in gas.

(f) (1 point) On the front page of your exam, CLEARLY write down your name and circle your lecture. In your scantron, use a pencil to bubble in your name, CORRECT netid, student number, your answers to questions 3–14, and your exam code.
Solution.

(a) (5 points) The consumption matrix for the part of the economy served by this company is:

\[
C = \begin{bmatrix}
0.7 & 0.35 \\
0.4 & 0.2 \\
\end{bmatrix}
\]

(b) (2 points) Interpretation of Column 1: To make $1.00 worth of oil, the company requires $1.1 in raw materials.

Interpretation of Row 2: In the production $1.00 of oil and $1.00 of gas, the company consumes $0.6 in gas.

(c) (2 points) Which sectors of the economy are profitable? Oil sector, Gas sector.

(d) (5 points) Find the matrices \( I - C \) and \( (I - C)^{-1} \).

\[
I - C = \begin{bmatrix}
0.3 & -0.35 \\
-0.4 & 0.8 \\
\end{bmatrix}
\]

\[
(I - C)^{-1} = \begin{bmatrix}
8 & 3.5 \\
4 & 3 \\
\end{bmatrix}
\]

Is the economy serviced by this company productive? YES, NO

(e) (5 points) Societal demands for what the Illinois Power Company produces are $100 for oil and $200 for gas. Is there a production schedule that will meet this demand? If no, enter NO in each of the following spaces. If yes, then the Illinois Power Company needs to produce $1500 in oil, $1000 in gas.

(f) (1 point) On the front page of your exam, CLEARLY write down your name and circle your lecture. In your scantron, use a pencil to bubble in your name, CORRECT netid, student number, your answers to questions 3–14, and your exam code.
3. (5 points) Given the matrices $A = \begin{bmatrix} 2 & 0 & 4 \\ 1 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & t \\ 2 & 0 & 1 \end{bmatrix}$, calculate $AB^\top$.

(A) $\begin{bmatrix} 8 & 2 & 2t + 1 \\ -4 & 0 & -2 \\ 12 & 4 & 4t \end{bmatrix}$

(B) $\begin{bmatrix} 5 & 1 & t + 4 \\ 3 & -2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 4t + 6 & 1 \\ 8 & 2 \end{bmatrix}$

(D) $\star \begin{bmatrix} 4t + 6 & 8 \\ 1 & 2 \end{bmatrix}$

(E) The matrices $A$ and $B^\top$ cannot be multiplied.

Solution. -
4. (5 points) For what value of $k$ are the vectors $(k, 7, 4)$ and $(1, k, -4)$ orthogonal (i.e., perpendicular)?

(A) 7
(B) 1
(C) ★ 2
(D) 0
(E) −2

Solution. -
5. (5 points) The graphs below (drawn on the same scales) show two given vectors $\vec{u}$ and $\vec{v}$ and four other vectors.

Which of $\vec{p}$, $\vec{q}$, $\vec{r}$ and $\vec{s}$, if any, is the vector $-2\vec{u} + \vec{v}$?

(A) None of these.
(B) $\vec{s}$
(C) $\vec{r}$
(D) $\star \vec{q}$
(E) $\vec{p}$

Solution. -
6. (5 points) *Thai Flies When You’re Having Fun*, a Thai restaurant, wants to make three different kinds of curry from a supply of three ingredients: coconut milk, chicken and potatoes, and wishes to maximize its profit. This leads to the following initial table for the simplex algorithm:

\[
\begin{bmatrix}
1 & -60 & -70 & -80 & 0 & 0 & 0 & 0 \\
0 & 8 & 4 & 10 & 1 & 0 & 0 & 500 \\
0 & 8 & 1 & 2 & 0 & 1 & 0 & 40 \\
0 & 4 & 4 & 4 & 0 & 0 & 1 & 140 \\
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>oz coconut milk</th>
<th>oz chicken</th>
<th>oz potatoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-60</td>
<td>-70</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

If the final simplex table for this problem is

\[
\begin{bmatrix}
1 & 80 & 0 & 0 & 0 & 10 & 15 & 2500 \\
0 & -38 & 0 & 0 & 1 & -6 & 1/2 & 330 \\
0 & 7 & 0 & 1 & 0 & 1 & -1/4 & 5 \\
0 & -6 & 1 & 0 & 0 & -1 & 1/2 & 30 \\
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>oz coconut milk</th>
<th>oz chicken</th>
<th>oz potatoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-38</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
<td>1</td>
</tr>
</tbody>
</table>

then

(A) 5 oz of chicken and 30 oz of potatoes are unused, and all of the coconut milk is used up.

(B) All of the ingredients are all used up.

(C) 2500 oz of ingredients remain.

(D) 5 oz of chicken, 30 oz of potatoes and 330 oz of coconut milk are unused.

(E) ★ 330 oz of coconut milk is unused and the rest of the ingredients are all used up.

Solution. -
7. (5 points) A manufacturer of bicycles has three factories which produce every day, in an eight hour shift, bicycles according to this table:

<table>
<thead>
<tr>
<th></th>
<th>road bike</th>
<th>hybrid bike</th>
<th>mountain bike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factory A</td>
<td>60</td>
<td>120</td>
<td>60</td>
</tr>
<tr>
<td>Factory B</td>
<td>30</td>
<td>45</td>
<td>40</td>
</tr>
<tr>
<td>Factory C</td>
<td>90</td>
<td>90</td>
<td>80</td>
</tr>
</tbody>
</table>

Is it possible to close Factory A and replace its production EXACTLY by running the Factories B and C overtime? If so, select the new productions schedule. If not, select the reason why not.

(A) Yes. To replace Factory A, we must run Factory B for a total of 16 hours a day and Factory C for a total of 12 hours a day.

(B) Yes. To replace Factory A, we must run Factory B for a total of 12 hours a day and Factory C for a total of 10 hours a day.

(C) ★ No. It is impossible to express the production vector of Factory A as a linear combination of the production vectors of Factory B and Factory C.

(D) No. We would have to run either Factory B or Factory C for more than 24 hours, which is impossible.

Solution. -
8. (5 points) The graph below shows three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ of the same length.

Of these three vectors, for which two vectors is the dot product smallest?

(A) $\vec{v} \cdot \vec{w}$ is the smallest.
(B) $\vec{u} \cdot \vec{w}$ is the smallest.
(C) $\vec{u} \cdot \vec{v}$ is the smallest.
(D) They are all the same.

Solution. -
9. (5 points) For what values of \( m \) and \( n \) are the vectors \((-3, 2, m)\) and \((n, -8, 4)\) parallel?

(A) The vectors are never parallel.
(B) \( m = -4, n = 3 \)
(C) \( m = 4, n = -3 \)
(D) \( m = -1, n = 12 \)
(E) \( m = 1, n = -12 \)

Solution. -
10. (5 points) The matrix $M = \begin{bmatrix} 3 & t \\ -6 & 4 \end{bmatrix}$ has an inverse when $t \neq -2$. Which of the following matrices is $M^{-1}$?

(A) $\begin{bmatrix} \frac{1}{6t+12} & 4-t \\ 6 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} -3 & -6 \\ t & -4 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{6t+12} & -3 \\ t & -6 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & -t \\ 6 & 3 \end{bmatrix}$

(E) $\begin{bmatrix} 1 \\ 6t+12 \end{bmatrix}$

Solution. -
11. (5 points) What is the length of vector that starts at the point \((3, 2, -1)\) and ends at the point \((5, 0, 3)\)?

(A) 24
(B) 4
(C) \(\sqrt{8}\)
(D) 8
(E) \(\star \sqrt{24}\)

Solution. -
12. (5 points) Let $A$, $B$, and $C$ be invertible $3 \times 3$ matrices. Consider the statements:

(I) $(BC)^{-1} = B^{-1}C^{-1}$.

(II) If $CA = CB$, then $A = B$.

(III) The RREF of $A$ is the identity matrix.

Which of the above statements are true (for every choice of invertible $3 \times 3$ matrices $A, B, C$)?

(A) ★ Only (II) and (III) are true.

(B) Only (I) is true.

(C) Only (I) and (II) are true.

(D) Only (I) and (III) are true.

(E) (I), (II), and (III) are all true.

Solution. -
13. (5 points) A bakery makes 3 types of muffins: blueberry, lemon, and chocolate at two locations: North Side and South Side. The North Side bakery produces 8 batches of blueberry muffins, 7 batches of lemon muffins, and 8 batches of chocolate muffins a day. The South Side factory produces 6 batches of blueberry muffins, 6 batches of lemon muffins, and 4 batches of chocolate muffins a day. The wholesale (W) and retail (R) prices for a batch of each item are $13 and $19 for blueberry muffins, $22 and $33 for lemon muffins, and $18 and $27 for chocolate muffins.

Suppose that the stock matrix (S) and price matrix (P) have the following forms:

\[
S = \begin{bmatrix}
\text{blueberry} & \text{lemon} & \text{chocolate} \\
\text{North} & \text{South} \\
\end{bmatrix} \quad P = \begin{bmatrix}
\text{blueberry} & \text{lemon} & \text{chocolate} \\
\text{W} & \text{R} \\
\end{bmatrix}
\]

Which matrix product should you use to find the total wholesale and retail value of the products at each location?

(A) \(SP\)
(B) \(\star SP^T\)
(C) The correct answer is not here.
(D) \(S^TP\)
(E) \(S^TP^T\)

Solution. -
14. (5 points) Which of the following is a vector equation of the line through the points \((-4, 1, -9)\) and \((5, 4, 5)\)?

(A) \(\vec{x} = (5, 4, 5) + t(-4, 1, -9)\)

(B) None of these is an equation of the line.

(C) \(\star \vec{x} = (-4, 1, -9) + t(9, 3, 14)\)

(D) \(\vec{x} = (-4, 1, -9) + t(5, 4, 5)\)

(E) \(\vec{x} = (9, 3, 14) + t(-4, 1, -9)\)

Solution. -