DO NOT OPEN EXAM UNTIL TOLD TO DO SO
1. (20 points) ________________

2. (20 points) ________________

3. (5 points) ________________

4. (5 points) ________________

5. (5 points) ________________

6. (5 points) ________________

7. (5 points) ________________

8. (5 points) ________________

9. (5 points) ________________

10. (5 points) ________________

11. (5 points) ________________

12. (5 points) ________________

13. (5 points) ________________

14. (5 points) ________________

TOTAL (100 points) ____________

92. B
93. A
94. A
95. E
96. D
1. (20 points) The company ‘Happy Garden’ builds garden sheds. A large shed requires 90 wall panels and 30 studs, and a small shed requires 30 wall panels and 20 studs. The company has available 270 wall panels and 120 studs. If they sell a large shed for $500 and a small shed for $300, how many of each type of building should they build to maximize their revenue?

Assign variables:

Define the objective function:

List all constraints:

Solve the linear program:

The maximum revenue of _____ dollars is attained by building _____ large sheds and _____ small sheds.
2. (20 points) You are organizing a dinner for your friends. You decide to make three types of tacos: regular, vegetarian, and supreme. One regular taco uses 2 oz of meat and 2 oz of beans. One vegetarian taco uses 0 oz of meat and 3 oz of beans. One supreme taco uses 4 oz of meat and 1 oz of beans. In your fridge, you have 14 oz of meat and 5 oz of beans. How many tacos of each type should you make if you intend to use all the meat and beans that you have available?

Assign variables:

The linear system to be solved is:

The RREF of the augmented matrix of this system is (you may use a calculator):

The parametric solution of the system is:

In order for all variables in this solution to have non-negative values, the parameter must lie in the interval:

How many tacos of each type should you make? Remember that you cannot make a fraction of a taco.

regular tacos: _____  vegetarian tacos: _____  supreme tacos: _____
3. (5 points) For the simplex table below, what is the basic feasible solution?

\[
\begin{array}{cccccc|c}
z & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\
1 & 3 & 0 & 2 & 0 & 9 & 0 & 22 \\
0 & 4 & 0 & 9 & 0 & 3 & 1 & 4 \\
0 & 1 & 1 & 5 & 0 & 7 & 0 & 6 \\
0 & 4 & 0 & 0 & 1 & 3 & 0 & 3 \\
\end{array}
\]

(A) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 6, 0, 3, 0, 4)\)

(B) \((x_1, x_2, x_3, s_1, s_2, s_3) = (3, 0, 2, 0, 9, 0)\)

(C) \((x_1, x_2, x_3, s_1, s_2, s_3) = (4, 6, 3, 0, 0, 0)\)

(D) \((x_1, x_2, x_3, s_1, s_2, s_3) = (4, 0, 6, 0, 3, 0)\)

(E) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 4, 0, 6, 0, 3)\)
4. (5 points) Reduced row echelon form (RREF) of a matrix is unique.

(A) True
(B) False
5. (5 points) Which of the following matrices are in row echelon form (REF)?

\[ M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 3 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

(A) All of them - \( M, N, \) and \( P \)

(B) Only \( N \) and \( P \)

(C) Only \( N \)

(D) None of the matrices are in row echelon form
6. (5 points) The row echelon form of the augmented matrix of a linear system is shown here:

\[
\begin{bmatrix}
1 & 0 & 4 & a \\
0 & 1 & 1 & b \\
0 & 0 & 1 & c \\
\end{bmatrix}
\]

The linear system is in the variable \(x\), \(y\) and \(z\), and \(a\) and \(b\) are real numbers. Express the solution for \(x\), \(y\) and \(z\) in terms of \(a\) and \(b\).

(A) The correct solution is not here.

(B) \(x = a - 4c\), \(y = b - c\), \(z = c\)

(C) \(x = a + 4b\), \(y = b + c\), \(z = c\)

(D) \(x = 4a\), \(y = b\), \(z = 0\)
7. (5 points) What is the geometric shape of this solution set?

\[ \{(t + 3, 2t - 5, t) \mid t \in \mathbb{R}\} \]

(A) The empty set.
(B) A single point.
(C) The correct answer is not here.
(D) A plane.
(E) A line.
8. (5 points) Which of the following statements are true?

I. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\] is 2.

II. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix}
\] is 3.

III. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\] is 3.

(A) Only I and III are true.

(B) Only I is true.

(C) Only I and II are true.

(D) All of the above are true.
9. (5 points) Consider the following four statements about a homogeneous linear system:

(I) may have no solution

(II) may have just one solution

(III) may have exactly two solutions

(IV) may have infinitely many solutions

Which of these possibilities are true?

(A) Only I, II, and III

(B) Only I and IV

(C) All of the statements are true

(D) Only II and IV

(E) Only I, II, and IV
10. (5 points) The picture below shows a feasibility region of a linear program in which we are to minimize some cost function \( C \). The cost function \( C \) is constant along each line of constancy \( \ell_1 \) and \( \ell_2 \). If \( C = 30 \) along the line \( \ell_1 \) and \( C = 20 \) along the line \( \ell_2 \), then the minimum cost occurs at the corner:

(A) \( P_1 \)
(B) \( P_3 \)
(C) \( P_5 \)
(D) \( P_4 \)
(E) \( P_2 \)
11. (5 points) Which of the following matrices are in reduced row echelon form (RREF)?

\[ M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(A) None of the matrices are in reduced row echelon form  
(B) Only \(N\)  
(C) Only \(M\) and \(N\)  
(D) Only \(M\)
12. (5 points) Find the value for $k$ that makes the following system of equations INCONSISTENT.

\[
\begin{align*}
3x + 3z &= 9 \\
4x + y + 3z &= 1 \\
6x - y + k \cdot z &= 10
\end{align*}
\]

(A) Only $k = 7$ makes the system inconsistent.
(B) Only $k = 19$ makes the system inconsistent.
(C) There is more than one value of $k$ that makes the system inconsistent.
(D) Only $k = 8$ makes the system inconsistent.
13. (5 points) Which of these graphs is the feasibility region for the following system of linear inequalities?

\[
\begin{align*}
5x + 4y &\leq 20 \\
2x + 5y &\leq 10 \\
x, y &\geq 0
\end{align*}
\]

(A) Graph I.
(B) Graph II.
(C) Graph IV.
(D) Graph III.
14. (5 points) Consider the system of equations below:

\[
\begin{align*}
3x - 2y + 4z + 5w &= -17 \\
-x + 2y - 3z - 2w &= 21 \\
5x - 2y + 2z + 5w &= -7 \\
x, y, z, w, \geq 0
\end{align*}
\]

\[
\begin{bmatrix}
3 & -2 & 4 & 5 & -17 \\
-1 & 2 & -3 & -2 & 21 \\
5 & -2 & 2 & 5 & -7
\end{bmatrix}
\xrightarrow{RREF}
\begin{bmatrix}
1 & 0 & 0 & 1 & 3 \\
0 & 1 & 0 & 1 & 9 \\
0 & 0 & 1 & 1 & -2
\end{bmatrix}
\]

Assuming you only want to consider non-negative solutions which are integers, how many solutions does this system have?

(A) Infinitely many solutions

(B) 6 solutions

(C) No solutions

(D) 4 solutions