Math 125 Fall 2017 Exam 1
Monday, October 2, 2017

Name: ___________________________  NetID: ___________________________@illinois.edu

Signature: ___________________________

Circle your lecture:

8am lecture  2pm lecture  3pm lecture

• No cells phones, i-Pods, MP3 players. Turn them off now. If you are seen these items in hand during the exam it will be considered cheating and you will be asked to leave.

• Print your proper name and NetID clearly at the top of this page, sign the exam below your name, and circle the section for which you are registered.

• There are 14 question in the exam, 2 of which are free response and 12 are multiple choice.

• In the free response part, you must show all your work to receive full credit.

• In the multiple choice part, you must circle your final choice with a regular pen (not a pencil!). Showing your work is NOT required for the multiple choice part.

• No questions will be permitted during the exam. If you are uncertain about something, put a small note about it on your exam paper and then answer the question as best you can with the information that you are given.

• You have 60 minutes to complete this exam.

• If you finish early, quietly and respectfully get up and hand in your exam.

• When time is up, put down your writing utensil, close your exam, and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

Calculator approved?  YES  NO
1. (20 points) ________________

2. (20 points) ________________

3. (5 points) ________________

4. (5 points) ________________

5. (5 points) ________________

6. (5 points) ________________

7. (5 points) ________________

8. (5 points) ________________

9. (5 points) ________________

10. (5 points) ________________

11. (5 points) ________________

12. (5 points) ________________

13. (5 points) ________________

14. (5 points) ________________

**TOTAL (100 points) ________________**

92. A
93. A
94. A
95. D
96. C
1. (20 points) The company ‘Organize It!’ builds tool sheds. A small shed requires 20 sheets of dry wall and 30 studs, and a large shed requires 30 sheets of dry wall and 90 studs. The company has available 120 sheets of dry wall and 270 studs. If they sell a small shed for $600 and a large shed for $1000, how many of each type of building should they build to maximize their revenue?

Assign variables:

Define the objective function:

List all constraints:

Solve the linear program:

The maximum revenue of $ \_\_\_\_\_\_ is attained by building \_\_\_\_ small sheds and \_\_\_\_ large sheds.
2. (20 points) You are organizing a dinner for your friends. You decide to make three types of tacos: regular, vegetarian, and supreme. One regular taco uses 2 oz of meat and 2 oz of beans. One vegetarian taco uses 0 oz of meat and 3 oz of beans. One supreme taco uses 4 oz of meat and 1 oz of beans. In your fridge, you have 14 oz of meat and 5 oz of beans. How many tacos of each type should you make if you intend to use all the meat and beans that you have available?

Assign variables:

The linear system to be solved is:

The RREF of the augmented matrix of this system is (you may use a calculator):

The parametric solution of the system is:

In order for all variables in this solution to have non-negative values, the parameter must lie in the interval:

How many tacos of each type should you make? Remember that you cannot make a fraction of a taco.

regular tacos: ______  vegetarian tacos: ______  supreme tacos: ______
3. (5 points) Which of these graphs is the feasibility region for the following system of linear inequalities?

\[
\begin{align*}
5x + 4y & \geq 20 \\
2x + 5y & \leq 10 \\
x, y & \geq 0
\end{align*}
\]

(A) Graph III.  
(B) Graph I.  
(C) Graph II.  
(D) Graph IV.
4. (5 points) For the simplex table below, what is the basic feasible solution?

\[
\begin{bmatrix}
    z & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\
    1 & 3 & 0 & 2 & 0 & 9 & 0 & 22 \\
    0 & 4 & 0 & 9 & 0 & 3 & 1 & 4 \\
    0 & 1 & 1 & 5 & 0 & 7 & 0 & 6 \\
    0 & 4 & 0 & 0 & 1 & 3 & 0 & 3 \\
\end{bmatrix}
\]

(A) \((x_1, x_2, x_3, s_1, s_2, s_3) = (4, 6, 3, 0, 0, 0)\)
(B) \((x_1, x_2, x_3, s_1, s_2, s_3) = (4, 0, 6, 0, 3, 0)\)
(C) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 4, 0, 6, 0, 3)\)
(D) \((x_1, x_2, x_3, s_1, s_2, s_3) = (3, 0, 2, 0, 9, 0)\)
(E) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 6, 0, 3, 0, 4)\)
5. (5 points) Which of the following statements are true?

I. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\] is 2.

II. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] is 3.

III. The row rank of \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\] is 3.

(A) All of the above are true.
(B) Only I and II are true.
(C) Only I and III are true.
(D) Only I is true.
6. (5 points) What is the geometric shape of this solution set?

\[ \{(2s + t, s, t) \mid t, s \in \mathbb{R}\} \]

(A) The correct answer is not here.

(B) A plane.

(C) A line.

(D) The empty set.

(E) A single point.
7. (5 points) The picture below shows a feasibility region of a linear program in which we are to maximize some revenue function $R$. The revenue function $R$ is constant along each line of constancy $\ell_1$ and $\ell_2$. If $R = 30$ along the line $\ell_1$ and $R = 20$ along the line $\ell_2$, then the maximum revenue occurs at the corner:

(A) $P_5$
(B) $P_1$
(C) $P_3$
(D) $P_2$
(E) $P_4$
8. (5 points) Find the value for $k$ that makes the following system of equations INCONSISTENT.

\begin{align*}
3x + 3z &= 9 \\
4x + y + 3z &= 1 \\
6x - y + k \cdot z &= 10
\end{align*}

(A) There is more than one value of $k$ that makes the system inconsistent.
(B) Only $k = 19$ makes the system inconsistent.
(C) Only $k = 7$ makes the system inconsistent.
(D) Only $k = 8$ makes the system inconsistent.
9. (5 points) Consider the following four statements about a homogeneous linear system:

(I) may have infinitely many solutions
(II) may have exactly two solutions
(III) may have just one solution
(IV) may have no solution

Which of these possibilities are true?

(A) Only I and IV
(B) Only I, III, and IV
(C) Only II, III, and IV
(D) All of the statements are true
(E) Only I and III
10. (5 points) Which of the following matrices are in reduced row echelon form (RREF)?

\[ M = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 4 & 2 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]

(A) None of the matrices are in reduced row echelon form

(B) Only \( N \)

(C) Only \( M \) and \( N \)

(D) Only \( M \)
11. (5 points) Which of the following matrices are in row echelon form (REF)?

\[ M = \begin{bmatrix}
1 & 0 & 1 & 5 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad N = \begin{bmatrix}
1 & 3 & 0 & 6 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 \\
0 & 1 & 4 & 0
\end{bmatrix}, \quad P = \begin{bmatrix}
1 & 0 & 2 & 6 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix} \]

(A) None of the matrices are in row echelon form
(B) Only \( M \)
(C) Only \( M \) and \( N \)
(D) Only \( M \) and \( P \)
12. (5 points) Reduced row echelon form (RREF) of a matrix is unique.

(A) True
(B) False
13. (5 points) The row echelon form of the augmented matrix of a linear system is shown here:

\[
\begin{bmatrix}
1 & 0 & 4 & a \\
0 & 1 & 1 & b \\
0 & 0 & 1 & c
\end{bmatrix}
\]

The linear system is in the variable \(x\), \(y\) and \(z\), and \(a\) and \(b\) are real numbers. Express the solution for \(x\), \(y\) and \(z\) in terms of \(a\) and \(b\).

(A) The correct solution is not here.
(B) \(x = 4a, y = b, z = 0\)
(C) \(x = a + 4b, y = b + c, z = c\)
(D) \(x = a - 4c, y = b - c, z = c\)
14. (5 points) Consider the system of equations below:

\[
\begin{align*}
3x - 2y + 4z + 5w &= -17 \\
-x + 2y - 3z - 2w &= 21 \\
5x - 2y + 2z + 5w &= -7 \\
x, y, z, w, \geq 0
\end{align*}
\]

Assuming you only want to consider non-negative solutions which are integers, how many solutions does this system have?

(A) No solutions
(B) 4 solutions
(C) Infinitely many solutions
(D) 6 solutions