1) You should have already checked into the exam and had your calculator approved. No exam will be accepted from a student who does not check in before they start the exam.

2) No hats or dark sunglasses. All hats are to be removed.

3) All book bags should be closed and placed under your seat. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out NOW.

4) No cells phones. Turn them off now. If you are seen with a cell phone in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece. No iPods or MP3s players, etc. Same rules as with cell phones.

5) Check to see that you have all of the pages. Each exam has a total of 40 problems.

6) No questions will be permitted during the exam.

7) If you finish early, quietly and respectfully get up and hand in your exam.

8) Good luck. You have 3 hours to complete the exam.

Instructions for Entering Multiple Choice Answers:

On this exam you will be entering your answers to the multiple choice questions on a scantron sheet that is included with your exam. Always use a Number 2 pencil, maintaining to shade darkly within the bubble. Do NOT cross out your mistakes, but rather erase them thoroughly before entering another answer. Before beginning, please code in your name, UIN, netID and FORM in the appropriate places. In addition, on your exam paper circle the choices you made on the scantron sheet in case there are difficulties in reading your sheet electronically.

Final Important Directions:

1) You must turn in this exam paper with your scantron sheet at the end of the exam, otherwise you will receive a score of zero!

2) You will lose 10 points on the exam if your name or netID or UIN or FORM are NOT bubbled in on your scantron sheet!
1. (1 point) A linear system

\begin{align*}
3x + y + 4z &= a \\
x + 2y + 3z &= b \\
4x + 3y + 7z &= c
\end{align*}

is consistent if

A none of the variables is a slack variable
B at least one of the variables is a free variable
C the linear system has a solution
D there is no solution of the system

2. (1 point) A paper company produces printer paper and construction paper at a pair of factories in Hammond and Atlanta. They have recently received an order for 360 cases of printer paper and 300 cases of construction paper to be filled within the next 21 days. They can produce 30 cases of printer paper and 15 cases of construction paper per day at a cost of $1000 at the Hammond factory and they can produce 60 cases of printer paper and 60 cases of construction paper per day at a cost of $3000 at the Atlanta factory. Characterize how the company should fill the order at a minimize cost.

This is a word problem of the following type:

A finding how to reduce the number of factories a company needs to run
B solving an optimization problem
C finding the number of days of production required to satisfy a contract
D showing how to make one product as a linear combination of others

3. (1 point) The set of solutions of

\begin{align*}
3x + y + 4z &= 0 \\
x + 2y + 3z &= 0 \\
4x + 3y + 7z &= 0
\end{align*}

in three dimensional space is:

A a set of points in the positive octant
B the intersection of three planes
C the intersection of three lines
D there are no solutions

4. (1 point) A Markov Chain model is useful for examining:

A trends in market shares over time
B how a company can optimize profits
C whether or not an economy can meet public demand for products
D whether a company can satisfy the requirements of a contract for products

5. (1 point) Which one of the following properties of matrices $A$, $B$ and $C$ is NOT generally true (assuming that all sums and products make sense)?

A $A + B = B + A$
B $(A + B)C = AC + BC$
C $7AB = A(7B)$
D If $AC = BC$, then $A = B$
6. (1 point) A Leontif model is useful for examining:
   A trends in market shares over time
   B how a company can optimize profits
   C whether or not an economy can meet public demand for products
   D whether a company can satisfy the requirements of a contract for products

7. (1 point) Gaussian Elimination is a procedure for:
   A finding the basic feasible solution of a simplex table
   B finding the inverse of a matrix
   C pivoting on an entry in a matrix
   D reducing a matrix to row echelon form using elementary row operations

8. (1 point) When a collection of vectors is linearly dependent, then:
   A there is a non-trivial linear combination of them that makes the zero vector
   B one is a linear combination of the others
   C at least one vector in the set can be eliminated without changing their span
   D all of the above

9. (1 point) A subset of \( \mathbb{R}^n \) is a subspace if:
   A every member of the subset has an additive inverse
   B it contains the zero vector \( \vec{0} \)
   C it is closed under forming linear combinations
   D scalar multiplication distributes over vector addition

10. (1 point) Which of the following collections is NOT an orthonormal basis of \( \mathbb{R}^4 \)?
    A the rows of the 4 \( \times \) 4 identity matrix
    B the vectors \( \vec{v}_1 = \left( \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \vec{v}_2 = \left( \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right), \vec{v}_3 = (0, 1, 0) \)
    C the columns of the 4 \( \times \) 4 identity matrix
    D the vectors \( \vec{v}_1 = \left( \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0 \right), \vec{v}_2 = \left( \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0 \right), \vec{v}_3 = (0, 1, 0, 0), \vec{v}_4 = (0, 0, 0, 1) \)

11. A simple economy has two industries: food and labor. The production of $1 in food requires 90\(c\) in food and 10\(c\) in labor, while the production of $1 in labor requires 20\(c\) in food and 60\(c\) in labor? There is an external demand for $100 in food and $200 in labor. To meet this demand, the economy must produce
    A $4000 in food and $1500 in labor
    B -$4000 in food and -$1500 in labor
    C $3000 in food and $2000 in labor
    D $2000 in food and $3000 in labor
12. Which of the regions in the figure below is the feasibility region of the system of linear inequalities

\[
\begin{align*}
  x + 2y & \geq 6 \\
  2x + y & \geq 6 \\
  x & \geq 0 \\
  y & \geq 0
\end{align*}
\]

A Region 1  
B Region 2  
C Region 3  
D Region 4
13. The minimum of \( z = 14x + 10y \) over the feasibility region

occurs at
A the point \((1, 2)\)
B the point \((2, 1)\)
C the point \((0, 4)\)
D there is no minimum
14. The corners of the feasibility region of the system

\[
\begin{align*}
2x + y & \geq 6 \\
x + y & \leq 6 \\
x + 2y & \geq 6 \\
x & \leq 7 \\
y & \leq 7
\end{align*}
\]

(here is a grid in case you want to do some plotting)

\[
\begin{array}{|c|c|c|}
\hline
 & 2 & \\
\hline
2 & & \\
\hline
\end{array}
\]

are

A \quad (0, 6), (2, 2), (6, 0)
B \quad (0, 6), (1, 4), (2, 2), (4, 1), (6, 0)
C \quad (0, 0), (0, 3), (2, 2), (3, 0)
D \quad (0, 7), (0, 6), (2, 2), (6, 0), (7, 0), (7, 7)

15. After one pivot in the simplex method for the linear program

maximize \quad z = -15x_1 - 14x_2 + 24x_3 \\
subject to \quad 4x_1 - 2x_2 + 2x_3 \leq 2 \\
16x_1 - 2x_2 + 4x_3 \leq 10 \\
15x_1 - 5x_2 + 6x_3 \leq 15 \\
x_1 \geq 0, x_2 \geq 0, x_3 \geq 0

the simplex table is

A: \quad \begin{bmatrix}
1 & -15 & -14 & 24 & 0 & 0 & 0 & 0 \\
0 & 4 & -2 & 2 & 1 & 0 & 0 & 2 \\
0 & 16 & -2 & 4 & 0 & 1 & 0 & 10 \\
0 & 15 & -5 & 6 & 0 & 0 & 1 & 15
\end{bmatrix}

B: \quad \begin{bmatrix}
1 & 103 & 0 & 0 & 2 & 5 & 0 & 54 \\
0 & 6 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 & 4 \\
0 & 4 & 1 & 0 & -1 & \frac{1}{2} & 0 & 3 \\
0 & -1 & 0 & 0 & -2 & -\frac{1}{2} & 1 & 6
\end{bmatrix}

C: \quad \begin{bmatrix}
1 & 63 & -10 & 0 & 12 & 0 & 0 & 24 \\
0 & 2 & -1 & 1 & \frac{1}{2} & 0 & 0 & 1 \\
0 & 8 & 2 & 0 & -2 & 1 & 0 & 6 \\
0 & 3 & 1 & 0 & -3 & 0 & 1 & 9
\end{bmatrix}

D: \quad \begin{bmatrix}
1 & 0 & -\frac{43}{2} & 63 & 0 & 15 & 0 & 0 & \frac{15}{2} \\
0 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 6 & -4 & -4 & 1 & 0 & 2 \\
0 & 0 & \frac{5}{2} & -\frac{3}{2} & -\frac{15}{4} & 0 & 1 & \frac{15}{2}
\end{bmatrix}
16. Consider the following statements about a linear program:
   a) It is possible that the maximum is achieved at exactly one point of the feasibility region.
   b) It is possible that the maximum is achieved at exactly two points of the feasibility region.
   c) It is possible that the maximum is achieved at infinitely many points of the feasibility region.
   d) It is possible that there is no maximum.
Which statements are true?
   A All but b are true
   B Only a, b and d
   C All of them are true
   D Only a and d

17. Which of the matrices
   \[ M = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 5 \\
\end{bmatrix} \]
are in row echelon form?
   A Only M
   B None of them
   C Only M and N
   D All of them

18. Which of the matrices
   \[ M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\
\end{bmatrix} \]
are in reduced row echelon form?
   A Only N
   B None of them
   C Only M and N
   D All of them

19. For a certain linear system in variables \(x, y, \) and \(z\), the row echelon form of the augmented matrix is
   \[ \begin{bmatrix} 1 & 3 & 5 & a \\ 0 & 1 & 2 & b \\ 0 & 0 & 1 & b \end{bmatrix} \]
   for some real numbers \(a\) and \(b\). So what are \(x, y\) and \(z\) expressed in terms of \(a\) and \(b\)?
   A \(x = a - 2b, y = -b, z = b\)
   B \(x = a - 5b, y = 3b, z = b\)
   C \(x = a - 9b, y = 3b, z = b\)
   D \(x = a, y = b, z = b\)
20. What relation between $a, b$ and $c$ ensures that the linear system

\[
\begin{align*}
x + y + z &= a \\
x + y + 2z &= b \\
2x + 2y + 2z &= c
\end{align*}
\]

is consistent?
A $c - 2a = 0$
B $a + b - c = 0$
C $a = b, c = 2a$
D none of the above

21. The solution of the linear system

\[
\begin{align*}
x_1 + 2x_2 + x_4 &= 5 \\
2x_1 + 4x_2 + x_3 + 5x_4 &= 12 \\
3x_1 + 6x_2 - x_3 + x_4 &= 15
\end{align*}
\]

is
A $(-2s - t + 5, s, -3t + 2, t)$, $s$ and $t$ any real numbers
B $(-2s - t + 5, s, -3t + 3, t)$, $s$ and $t$ any real numbers
C $(3 - 2s, s, -4, 2)$, $s$ any real number
D $(1, 3, 2, -1)$

22. The vectors $(k, 2, 4)$ and $(k, 5k, -4)$ are orthogonal only when $k$ satisfies:
A $k^2 + 5k = 0$
B $k^2 + 10k - 16 = 0$
C $k^2 + 5k - 16 = 0$
D no value of $k$

23. Which of the following is the equation of the line through $(2, 2, 1, 3)$ and $(-2, 4, 7, 1)$?
A $x = (2, 2, 1, 3) + t(4, -2, -6, 2)$
B $x = (-2, 4, 7, 1) + t(-4, 2, 6, -2)$
C $x = (2, 2, 1, 3) + t(2, -1, -3, 1)$
D All of the above

24. Jan’s Copyshop monthly loses $\frac{1}{6}$ of its customers to Paul’s Dup-it, while Paul’s loses $\frac{1}{3}$ of its customers to Jan’s in the same period. What is the exact stable vector for this process?
A $S = \begin{bmatrix} \frac{1}{6} \\ \frac{2}{3} \end{bmatrix}$ Jan’s
B $S = \begin{bmatrix} .33 \\ .67 \end{bmatrix}$ Paul’s
C $S = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$ Jan’s
D $S = \begin{bmatrix} .667 \\ .333 \end{bmatrix}$ Paul’s

25. For the previous problem, how long does it take the process to stabilize (with 4 decimal accuracy)?
A between 0 and 6 months?
B between 7 months and 12 months?
C between 13 months and 18 months?
D more than 18 months?
26. Let 
\[ v = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \]

Then \( v \) is a linear combination of
A \( u_1 \) and \( u_2 \)
B \( u_1 \) and \( u_3 \)
C \( u_2 \) and \( u_3 \)
D None of the above

27. Let 
\[ M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ m & n \end{bmatrix}, \quad N = \begin{bmatrix} 3 \\ x \end{bmatrix}, \quad P = \begin{bmatrix} 4 \\ x \\ y \end{bmatrix}, \quad Q = \begin{bmatrix} 5 \\ 5 \\ 3 \\ z \end{bmatrix} \]

and consider the statements:

a: \( MN \cdot P \) is a scalar  
b: \( MN \) is a \( 4 \times 1 \) matrix  
c: \( Q^TQ \) is a \( 1 \times 2 \) matrix  
d: \( MM^T \) is a \( 3 \times 3 \) matrix

Which statements are true?  
A Only a and b  
B Only b  
C All of them are true  
D All but a are true

28. A slack variable
A Appears in a parametric solution and can take on any real value  
B Is a left over variable  
C Measures the degree to which an inequality is satisfied as an equality  
D Can take on a negative value

29. In a pilot marketing analysis for a new handyman drill, the demand for the drill at various prices is listed below. Find the least squares regression line that relates demand to price.

<table>
<thead>
<tr>
<th>Price (x)</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (y)</td>
<td>84</td>
<td>74</td>
<td>62</td>
<td>50</td>
</tr>
</tbody>
</table>

A \( y = -2.28x + 130.2 \)  
B \( y = 2.28x - 130.2 \)  
C \( y = -2.28x - 130.2 \)  
D \( y = 2.28x + 130.2 \)
30. Consider the four matrices

\[ M = \begin{bmatrix} a & b & 3a \\ 3 & 7 & 9 \\ 2 & 2 & 6 \end{bmatrix}, \quad N = \begin{bmatrix} 4 & 0 & 8 & 2 \\ 6 & 0 & 7 & 2 \\ 7 & 0 & 1 & 2 \\ 1 & 0 & 1 & 7 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & a & b \\ 0 & 2 & 7 \\ 0 & 0 & 8 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

Which of these, if any, has a zero determinant?
A. \( N \) and \( Q \)
B. \( M, N \) and \( Q \)
C. \( M \) and \( N \)
D. all have determinant 0

31. Which of the following conditions on \( a, b, \) and \( c \) ensures that the following matrix is not invertible?

\[ M = \begin{bmatrix} 2 & a & 1 \\ 1 & b & -3 \\ -1 & c & 4 \end{bmatrix} \]

A. \(-a + 9b + 5c = 0\)
B. \(a - 9b - 7c = 0\)
C. \(-a + 7b + 5c = 0\)
D. \(-7a + 9b + 5c = 0\)

32. Eastbrook Quarries mines fine sand, coarse sand and gravel at three quarries. Quarry 1 produces 20 tons fine, 40 tons coarse and 20 tons gravel per day. Quarry 2 produces 60 tons fine, 60 tons coarse and 30 tons gravel per day. And Quarry 3 produces 30 tons fine, 40 tons coarse and 20 tons gravel per day. None of the quarries is worked every day. Quarry 3 can be closed and its daily output replaced with

A. running Quarry 1 for 1 additional day and Quarry 2 for \( \frac{1}{2} \) additional day
B. running Quarry 1 for \( \frac{1}{3} \) additional day and Quarry 2 for \( \frac{1}{2} \) additional day
C. running Quarry 1 for \( \frac{1}{2} \) additional day and Quarry 2 for 1 additional day
D. running Quarry 1 for \( \frac{1}{2} \) additional day and Quarry 2 for \( \frac{1}{3} \) additional day

33. Consider the following planes in \( \mathbb{R}^3 \):

\[ a \{(s, t, s + 3t) : s, t \in \mathbb{R}\} \]
\[ b \{(s + 3, t - 2, s + t + 1) : s, t \in \mathbb{R}\} \]
\[ c \{(3s - 4t, s + 2t, 1) : s, t \in \mathbb{R}\} \]
\[ d \{(2s + 4t, 0, s - t) : s, t \in \mathbb{R}\} \]

Which of them are subspaces of \( \mathbb{R}^3 \)?
A. Only \( a \) and \( c \)
B. All of them
C. All but \( c \)
D. Only \( a \) and \( d \)
34. Which of the following sets of vectors in $\mathbb{R}^3$ is linearly independent?

- a $\{(2,1,4), (-3,0,1), (0,0,0)\}$
- b $\{(4,1,3), (-5,0,3), (-1,1,2), (0,1,5)\}$
- c $\{(3,1,2), (-2,1,0), (1,2,1)\}$
- d $\{(1,0,2), (2,3,5)\}$

A Only c and d  
B All of them except b  
C Only d  
D All of them except a

35. Consider the truth of the statement below, assuming $A$ and $B$ are square matrices.

"If $\det(AB) \neq 0$, then both $A$ and $B$ are invertible."

Would the truth value of the statement change, if you are not told ahead of time that $A$ and $B$ are square matrices?

A The statement is true for square matrices and false for non-square matrices  
B The statement is true for square matrices and true for non-square matrices  
C The statement is false for square matrices and true for non-square matrices  
D The statement is false for square matrices and false for non-square matrices

36. If $\vec{u}$ is the normalization of a vector $\vec{v}$, then

A $\vec{u}$ is parallel to $\vec{v}$  
B $\vec{u}$ is shorter than $\vec{v}$  
C $\vec{u}$ is longer than $\vec{v}$  
D $\vec{u}$ is orthogonal to $\vec{v}$

37. Let $\vec{u} = (3,2,9)$ and $\vec{v} = (0,2,4)$. What is $\text{Proj}_v(\vec{u})$?

A $\text{Proj}_v(\vec{u}) = (0,4,8)$  
B $\text{Proj}_v(\vec{u}) = (0,1,2)$  
C $\text{Proj}_v(\vec{u}) = (0,8,4)$  
D $\text{Proj}_v(\vec{u}) = (0,-1,-2)$

38. Is the vector $\vec{v} = \langle -2,-1,1,-2 \rangle$ in the null space of matrix $A$?

$$A = \begin{bmatrix}
-2 & -3 & 3 & 5 \\
6 & 4 & 16 & 0 \\
4 & 3 & 9 & -1
\end{bmatrix}$$

A Yes, the vector is in the null space  
B No, the vector is not in the null space  
C The dimensions are incorrect and I cannot answer this question.
39. Is the vector $\vec{v} = < 3, -2, 5 >$ in the column space of matrix $A$?

$$A = \begin{bmatrix} -2 & -3 & 3 & 5 \\ 6 & 4 & 16 & 0 \\ 4 & 3 & 9 & -1 \end{bmatrix}$$

A No, the vector is not in the column space  
B Yes, the vector is in the column space  
C The dimensions are incorrect and I cannot answer this question.

40. Are the vectors $\vec{v}_1 = (2, 3, 5)$, $\vec{v}_2 = (-1, 2, -6)$, and $\vec{v}_3 = (1, 5, 1)$ linearly dependent? If so, what is a dependency equation?

A The set is not linearly dependent  
B The set is linearly dependent and there is a dependency equation given by $t\vec{v}_1 + t\vec{v}_2 + t\vec{v}_3 = 0$ for $t \neq 0$  
C The set is linearly dependent and there is a dependency equation given by $-t\vec{v}_1 - t\vec{v}_2 + t\vec{v}_3 = 0$ for $t \neq 0$  
D The set is linearly dependent and there is a dependency equation given by $0\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3 = 0$