For questions 1 - 5 below you should match each type of problem with 4 choices that apply to the solving of the problem. Each correct choice will be worth 1 point. Only the first 4 choices will be recorded. There may be more than four correct choices, and some choices can be used more than once. Please record your choices in the blanks at the right.

**CHOICES:**

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<tr>
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<th>a) A stochastic Matrix</th>
<th>c) ( \mathbf{x} = \mathbf{P}_0 + t\mathbf{v} )</th>
<th>i) ( \mathbf{u} \cdot \mathbf{v} )</th>
<th>n) ( \mathbf{x} = \mathbf{P}_0 + \text{Span}{\mathbf{v}_1, \mathbf{v}_2} )</th>
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<td></td>
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<td>d) ( \text{det}[A] )</td>
<td>j) ( [A \mid \mathbf{b}] )</td>
<td>o) Parametric solutions</td>
</tr>
<tr>
<td></td>
<td>c) A homogeneous system</td>
<td>g) ( \text{col}[A] )</td>
<td>k) ( T^n\mathbf{S}_0 )</td>
<td>p) An Identity Matrix</td>
</tr>
<tr>
<td></td>
<td>d) ( \mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_k\mathbf{v}_k )</td>
<td>h) Position vectors</td>
<td>m) ( \lVert \mathbf{v} \rVert )</td>
<td>q) ( \mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_k\mathbf{v}_k )</td>
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1) (4 points) A Markov Process Problem

2) (4 points) A Linear Combination Problem

3) (4 points) A Geometric Vector Problem

4) (4 points) A Subspace Problem

5) (4 points) A Linear Dependence Problem
3) A group of students on campus were surveyed about their eating habits and it was found they ate only burgers, pizza and enchiladas. The night the survey was taken 50% had eaten burgers, 30% had had pizza and 20% had eaten enchiladas. Their pattern was, if they ate burgers one night, 35% would again have a burger the next night, 25% would switch to pizza and 40% would switch to enchiladas. Of those who had pizza on any given night, 20% would have a burger the next night, 70% would stick with pizza and 10% would switch to enchiladas. Of those who had enchiladas on a given night, 20% would switch to burgers, 20% would switch to pizza and 60% would stay with enchiladas for the next night.

a) (3 points) Write a transition matrix for this situation. Be sure to clearly label the matrix.

\[
T = \begin{bmatrix}
0.35 & 0.20 & 0.40 \\
0.25 & 0.70 & 0.10 \\
0.40 & 0.10 & 0.60
\end{bmatrix}
\]

b) (2 points) Write and label the initial state matrix for this situation.

\[
S_0 = \begin{bmatrix}
0.50 \\
0.30 \\
0.20
\end{bmatrix}
\]

c) (1 point) What is the time period of each state in this situation?

\[
\text{1 Night (Day)}
\]

d) (4 points) Write an equation that will compute the percent of students eating each type of food after one week and interpret the solution. (Round your answers to the nearest percent)

\[
S_7 = T^7S_0
\]

\[
S_7 = \begin{bmatrix}
0.35 & 0.2 & 0.2 \\
0.25 & 0.7 & 0.2 \\
0.40 & 0.1 & 0.6
\end{bmatrix}
\begin{bmatrix}
0.50 \\
0.30 \\
0.20
\end{bmatrix}^7
\]

\[
= \begin{bmatrix}
0.42 \\
0.24 \\
0.34
\end{bmatrix}
\]

28% will eat burgers, 38% pizza, 35% enchiladas after 1 week.

e) (5 points) If this situation continues indefinitely, there will be an expected ratio of students who will be eating each type of food on any given night. What are those exact ratios? Show your augmented matrix for full credit.

\[
\begin{bmatrix}
-0.65 & 0.2 & 0 & 0 \\
0.25 & -3 & 0 & 1 \\
0 & 1 & 1 & 1
\end{bmatrix}
\text{rref}
\begin{bmatrix}
1 & 0 & 0 & 4/17 \\
0 & 1 & 0 & 36/85 \\
0 & 0 & 1 & 29/85
\end{bmatrix}
\]

4/17 will eat burgers, \( \frac{36}{85} \) pizza, \( \frac{29}{85} \) enchiladas.
7) Given \( A = \begin{bmatrix} 2 & -4 & -8 \\ 3 & -1 & 3 \\ -5 & 3 & -1 \end{bmatrix} \) \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad c_1 = -1 - 2t \\
\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad c_2 = -2 - 3t \\
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad c_3 = t 
\]

a) (5 points) Write a parametric equation that shows \((6, -1, -1)\) is a linear combination of the column vectors of \( A \).

\[
al \cdot 2 \vec{v}_1 + (-2 - 3t) \vec{v}_2 + t \vec{v}_3 
\]

b) (5 points) Are the column vectors of \( A \) linearly dependent? If so, write a parametric dependency equation.

\[
al \cdot 2 \vec{v}_1 - 3t \vec{v}_2 + t \vec{v}_3 
\]
8) For all real values of \( t, r, \) and \( s, \) let \( \mathbf{W} = \{(2t - r, 3t + 2s, 5s)\}, \) \( \mathbf{V} = \{(4t + s, 6 - 8s, -t + 2s)\}, \) and \( \mathbf{X} = \{(2t,4t),(1s,3s)\} \) be vector subsets.

a) (2 points) Which of these is not a subspace and why?

\[ \vec{\mathbf{W}} \text{ does not contain the zero vector} \]

b) (3 point) Which of these is a subset of \( \mathbb{R}^2 \) and how do you know?

\[ \vec{\mathbf{X}} \quad \text{each vector has 2 entries} \]

\[ (2\text{-space}) \]

\[ (\text{from } \mathbb{R}^2) \]

c) (5 points) Write the vectors of \( \mathbf{X} \) as a linear combination equation and describe its geometric interpretation.

\[ \mathbf{X} = t(2,4) + s(1,3) \]

A plane

d) (5 points) Write \( \mathbf{W} \) as a span.

\[ \mathbf{W} = \text{span} \left\{ (2,3,0), (-1,0,0), (0,2,5) \right\} \]
9 (2 points each) Mark each statement with “True” or “False” (not “T” or “F”)

a) True All vectors in a subspace of $\mathbb{R}^n$ will be n-tuples.

b) False All linear combinations are parametric.

c) True All homogeneous systems are consistent.

d) False All rows in a Markov analysis matrix add to 1.

e) False The dot product of two matrices in 3-space will be a vector in 3-space.

f) False Parallel vectors never meet.

g) False The magnitude of a vector can be negative.

h) True The span of a plane will have $\vec{0}$ as the initial point.

i) True The initial state of a Markov process does not affect the exact stable matrix.

j) False Vector multiplication is just like matrix multiplication.
10) The chart below shows the weekend work schedule at the local Starbucks. The shop would like to reduce the number of workers to 3, but must maintain the same total number of hours of operation.

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Saturday</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Sunday</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Putting this information in an augmented matrix in reduced echelon form yields:

\[
\begin{bmatrix}
1 & 0 & 0 & 3 & 0 \\
0 & 1 & 0 & -6 & 0 \\
0 & 0 & 1 & 2 & 0 \\
\end{bmatrix}
\]

\[
c_1 = -3t \\
c_2 = 6t \\
c_3 = -2t \\
c_4 = t
\]

a) (2 points) Write a parametric equation that shows the dependency among the workers.

\[
\vec{0} = -3t \vec{w}_1 + 6t \vec{w}_2 - 2t \vec{w}_3 + \vec{w}_4
\]

b) (4 points) Write a linear combination equation for each employee.

\[
w_1 = \frac{2}{5} \vec{w}_2 - \frac{2}{3} \vec{w}_3 + \frac{1}{3} \vec{w}_4
\]

\[
w_2 = \frac{1}{2} \vec{w}_1 + \frac{1}{3} \vec{w}_3 - \frac{1}{6} \vec{w}_4
\]

\[
w_3 = -\frac{3}{2} \vec{w}_1 + 3 \vec{w}_2 + \frac{1}{2} \vec{w}_4
\]

\[
w_4 = 3 \vec{w}_1 - 6 \vec{w}_2 + 2 \vec{w}_3
\]

b) (4 points) The store currently meets the operating hour schedule using the following formula:

\[
S = 5w_1 + 4w_2 + 4w_3 + 2w_4
\]

Can they find a schedule that allows them to eliminate one worker if no worker may work more than 10 hours a day? If so, state the new schedule, you need only find one.

\[
S_{w_1} = 14w_2 + \frac{2}{3}w_3 + \frac{11}{3}w_4 \quad \text{NO}
\]

\[
S_{w_2} = 7w_1 + 5\frac{1}{3}w_3 + \frac{4}{3}w_4 \quad \text{YES}
\]
1) The chart below shows the weekend work schedule at the local Starbucks. The shop would like to reduce the number of workers to 3, but must maintain the same total number of hours of operation.

<table>
<thead>
<tr>
<th>Workers</th>
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<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
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<td>4</td>
<td>6</td>
<td>0</td>
</tr>
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<td>4</td>
<td>8</td>
<td>4</td>
</tr>
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Putting this information in an augmented matrix in reduced echelon form yields:

\[
\begin{bmatrix}
1 & 0 & 0 & 3 & 0 \\
0 & 1 & 0 & -6 & 0 \\
0 & 0 & 1 & 2 & 0
\end{bmatrix}
\]

\[
\begin{align*}
\mathbf{c}_1 &= -3t \\
\mathbf{c}_2 &= 6t \\
\mathbf{c}_3 &= -2t \\
\mathbf{c}_4 &= t
\end{align*}
\]

a) (2 points) Write a parametric equation that shows the dependency among the workers.

\[
\begin{align*}
\mathbf{0} &= -3t \mathbf{w}_1 + (6t \mathbf{w}_2 - 2t \mathbf{w}_3 + t \mathbf{w}_4)
\end{align*}
\]

b) (4 points) Write a linear combination equation for each employee.

\[
\begin{align*}
\mathbf{w}_1 &= 2\mathbf{w}_2 - \frac{2}{3}\mathbf{w}_3 + \frac{1}{3}\mathbf{w}_4 \\
\mathbf{w}_2 &= \frac{1}{2}\mathbf{w}_1 + \frac{1}{3}\mathbf{w}_3 - \frac{1}{6}\mathbf{w}_4 \\
\mathbf{w}_3 &= -\frac{3}{2}\mathbf{w}_1 + 3\mathbf{w}_2 + \frac{1}{2}\mathbf{w}_4 \\
\mathbf{w}_4 &= 3\mathbf{w}_1 - 6\mathbf{w}_2 + 2\mathbf{w}_3
\end{align*}
\]

b) (4 points) The store currently meets the operating hour schedule using the following formula:

\[
S = 5\mathbf{w}_1 + 4\mathbf{w}_2 + 4\mathbf{w}_3 + 2\mathbf{w}_4
\]

Can they find a schedule that allows them to eliminate one worker if no worker may work more than 10 hours a day? If so, state the new schedule, you need only find one.

\[
\begin{align*}
\mathbf{S}_{\mathbf{w}_1} &= 14\mathbf{w}_2 + \frac{2}{3}\mathbf{w}_3 + \frac{4}{3}\mathbf{w}_4 \quad \text{NO} \\
\mathbf{S}_{\mathbf{w}_2} &= 7\mathbf{w}_1 + 5\frac{2}{3}\mathbf{w}_3 + \frac{2}{3}\mathbf{w}_4 \quad \text{YES}
\end{align*}
\]
For questions 2 - 6 below you should match each type of problem with 4 choices that apply to the solving of the problem. Each correct choice will be worth 1 point. Only the first 4 choices will be recorded. There may be more than four correct choices, and some choices can be used more than once. Please record your choices in the blanks at the right.

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<td>h) Position vectors</td>
<td>m) $</td>
<td>\vec{v}</td>
</tr>
<tr>
<td>$\vec{x} = c_1\begin{bmatrix} \vec{v}_1 \ \vec{v}_2 \ \vdots \ \vec{v}_k \end{bmatrix} + c_2\begin{bmatrix} \vec{v}_1 \ \vec{v}_2 \ \vdots \ \vec{v}_k \end{bmatrix} + \ldots + c_k\begin{bmatrix} \vec{v}_1 \ \vec{v}_2 \ \vdots \ \vec{v}_k \end{bmatrix}$</td>
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2) (4 points) A Geometric Vector Problem

3) (4 points) A Linear Dependence Problem

4) (4 points) A Subspace Problem

5) (4 points) A Markov Process Problem

6) (4 points) A Linear Combination Problem
7) Given \( A = \begin{bmatrix} 2 & -4 & -8 \\ 3 & -1 & 3 \\ -5 & 3 & -1 \end{bmatrix} \)

\[ \text{rref} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \]

\( c_1 = 2-2t \)
\( c_2 = 4-3t \)
\( c_3 = t \)

a) (5 points) Write a parametric equation that shows \((-12, 2, 2)\) is a linear combination of the column vectors of \( A \).

\((-12, 2, 2) = (2-2t)(2, 3, -5) + (4-3t)(-4, -1, 3) + t(-8, 3, 1)\)

OR

OTHER VERSIONS

b) (5 points) Are the column vectors of \( A \) linearly dependent? If so, write a parametric dependency equation.

\( \text{YES} \)

\[ \text{rref} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \]

\( c_1 = -2t \)
\( c_2 = -3t \)
\( c_3 = t \)

\[ 0 = -2t(2, 3, -5) - 3t(3, -1, 3) + t(-8, 3, 1) \]
8) For all real values of $t$, $r$, and $s$, let $\mathbf{W} = \{(4t + s, 6 - 8s, -t + 2s)\}$, $\mathbf{V} = \{(2t, 4t), (1s, 3s)\}$ and $\mathbf{X} = \{(2t - r, 3t + 2s, 5s)\}$ be vector subsets.

a) (2 points) Which of these is not a subspace and why?

\[ \mathbf{W} \text{ does not contain the zero vector} \]

b) (3 points) Which of these is a subset of $\mathbb{R}^2$ and how do you know?

\[ \mathbf{V} \text{ each vector has 2 entries} \]
\[ \mathbf{V} \text{ 2 dimensions} \]
\[ \mathbf{V} \text{ ordered pair} \]
\[ \mathbf{V} \text{ from } \mathbb{R}^2 \]

c) (5 points) Write $\mathbf{X}$ as a span.

\[ \mathbf{X} = \text{Span} \left\{ (2, 3, 0), (0, -1, 0), (0, 2, 5) \right\} \]

d) (5 points) Write the vectors of $\mathbf{V}$ as a linear combination equation and describe its geometric interpretation.

\[ \mathbf{V} = \pm (2, 4) + 5 (1, 3) \quad \text{A plane} \]
9 (2 points each) Mark each statement with “True” or “False” (not “T” or “F”)

a) **FALSE** Parallel vectors never meet.

b) **TRUE** All spans are parametric.

c) **FALSE** All multi-systems are consistent.

d) **TRUE** All vectors in a subspace of $\mathbb{R}^n$ will be n-tuples.

e) **FALSE** The dot product of two matrices in 3-space will be a vector in 3-space.

f) **FALSE** All rows in a Markov analysis matrix add to 1.

g) **TRUE** The magnitude of a vector cannot be negative.

h) **FALSE** The vector equation of a plane will always have $\vec{0}$ as the initial point.

i) **FALSE** The initial state of a Markov process determines the exact stable matrix.

j) **TRUE** Vector multiplication is not defined like matrix multiplication.
3) A group of students on campus were surveyed about their eating habits and it was found they ate only burgers, pizza and enchiladas. The night the survey was taken 50% had eaten burgers, 30% had had pizza and 20% had eaten enchiladas. Their pattern was, if they ate burgers one night, 35% would again have a burger the next night, 25% would switch to pizza and 40% would switch to enchiladas. Of those who had pizza on any given night, 20% would have a burger the next night, 70% would stick with pizza and 10% would switch to enchiladas. Of those who had enchiladas on a given night, 20% would switch to burgers, 20% would switch to pizza and 60% would stay with enchiladas for the next night.

a) (3 points) Write a transition matrix for this situation. Be sure to clearly label the matrix.

\[
T = \begin{bmatrix}
0.35 & 0.20 & 0.40 \\
0.25 & 0.70 & 0.10 \\
0.40 & 0.10 & 0.60
\end{bmatrix}
\]

\[
S_0 = \begin{bmatrix}
0.50 \\
0.30 \\
0.20
\end{bmatrix}
\]

b) (2 points) Write and label the initial state matrix for this situation.

c) (1 point) What is the time period of each state in this situation?

1 N I G H T ( D A Y )

\[
S_7 = T^7S_0
\]

\[
S_7 = \begin{bmatrix}
0.35 & 0.2 & 0.2 \\
0.25 & 0.7 & 0.1 \\
0.4 & 0.1 & 0.6
\end{bmatrix}^7
\]

\[
\begin{bmatrix}
0.5 \\
0.3 \\
0.2
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.24 \\
0.42 \\
0.32
\end{bmatrix}
\]

28% will eat burgers, 38% pizza, 35% enchiladas after 1 week.

d) (4 points) Write an equation that will compute the percent of students eating each type of food after one week and interpret the solution. (Round your answers to the nearest percent)

\[
28\% \ \text{will eat burgers, } 38\% \ \text{pizza, } 35\% \ \text{enchiladas after 1 week.}
\]

e) (5 points) If this situation continues indefinitely, there will be an expected ratio of students who will be eating each type of food on any given night. What are those exact ratios? Show your augmented matrix for full credit.

\[
\begin{bmatrix}
-0.65 & 0.2 & 0.2 & 0 \\
0.85 & -3 & 2 & 11 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 4/17 \\
0 & 1 & 0 & 36/85 \\
0 & 0 & 1 & 29/85
\end{bmatrix}
\]

\[
4/17 \ \text{will eat burgers, } 36/85 \ \text{pizza, } 29/85 \ \text{enchiladas}
\]
1) A group of students on campus were surveyed about their eating habits and it was found they ate only burgers, pizza and enchiladas. The night the survey was taken 50% had eaten burgers, 30% had had pizza and 20% had eaten enchiladas. Their pattern was, if they ate burgers one night, 35% would again have a burger the next night, 25% would switch to pizza and 40% would switch to enchiladas. Of those who had pizza on any given night, 20% would have a burger the next night, 70% would stick with pizza and 10% would switch to enchiladas. Of those who had enchiladas on a given night, 20% would switch to burgers, 20% would switch to pizza and 60% would stay with enchiladas for the next night.

a) (3 points) Write a transition matrix for this situation. Be sure to clearly label the matrix

\[ T = \begin{bmatrix}
B & P & E \\
0.35 & 0.2 & 0.2 \\
0.25 & 0.7 & 0.2 \\
0.4 & 1.1 & 0.6
\end{bmatrix} \]

b) (2 points) Write and label the initial state matrix for this situation.

\[ S_0 = \begin{bmatrix}
0.5 \\
0.3 \\
0.2
\end{bmatrix} \]

(c) (1 point) What is the time period of each state in this situation?

1 NIGHT, 1 DAY, ETC

d) (4 points) Write an equation that will compute the percent of students eating each type of food after one week and interpret the solution. (Round your answers to the nearest percent)

\[ S_7 = T^7 S_0 \]

\[ S_7 = \begin{bmatrix}
0.35 & 0.2 & 0.2 \\
0.25 & 0.7 & 0.2 \\
0.4 & 1.1 & 0.6
\end{bmatrix} \begin{bmatrix}
0.5 \\
0.3 \\
0.2
\end{bmatrix} = \begin{bmatrix}
0.24 \\
0.42 \\
0.34
\end{bmatrix} \]

24% EAT BURGERS, 42% PIZZA & 32% ENCHILADAS AFTER 1 WEEK

e) (5 points) If this situation continues indefinitely, there will be an expected ratio of students who will be eating each type of food on any given night. What are those exact ratios? Show your augmented matrix for full credit.

\[ (T-I) \begin{bmatrix}
-0.65 & 0.2 & 0.2 \\
0.25 & -0.3 & 0.2 \\
1 & 1 & 1
\end{bmatrix} \rightarrow \text{rref} \begin{bmatrix}
1 & 0 & 0 & \frac{4}{17} \\
0 & 1 & 0 & \frac{36}{85} \\
0 & 0 & 1 & \frac{29}{85}
\end{bmatrix} \]

\[ \frac{4}{17} \text{ WILL EAT BURGERS, } \frac{36}{85} \text{ WILL EAT PIZZA} \]

AND \[ \frac{29}{85} \text{ WILL EAT ENCHILADAS.} \]
2) Given \( A = \begin{bmatrix} 2 & -4 & -8 \\ 3 & -1 & 3 \\ -5 & 3 & -1 \end{bmatrix} \) \( \xrightarrow{rref} \) \( \begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \) \( c_1 = 1 - 2t \) \( c_2 = 2 - 3t \) \( c_3 = t \)

a) (5 points) Write a parametric equation that shows \((-6, 1, 1)\) is a linear combination of the column vectors of \( A \).

\[
\begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix} = (1 - 2t) \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} + (2 - 3t) \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -8 \\ 3 \\ -1 \end{bmatrix}
\]

\[
\begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix} = (1 - 2t) \vec{v}_1 + (2 - 3t) \vec{v}_2 + t \vec{v}_3
\]

IF LABEL THE ABOVE

YES

b) (5 points) Are the column vectors of \( A \) linearly dependent? If so, write a parametric dependency equation.

\[
\begin{bmatrix} 10 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \]

\( c_1 = -2t \) \( c_2 = -3t \) \( c_3 = t \)

\[
\begin{bmatrix} 0 \end{bmatrix} = -2t \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} - 3t \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -8 \\ 3 \\ -1 \end{bmatrix}
\]
3) (2 points each) Mark each statement with "True" or "False" (not "T" or "F")

a) **FALSE** Vector multiplication is defined as matrix multiplication

b) **TRUE** The initial state of a Markov process does not affect the exact stable matrix.

c) **FALSE** All homogeneous systems are parametric.

d) **TRUE** All columns in a Markov analysis matrix add to 1.

e) **FALSE** All linear combinations are parametric.

f) **FALSE** Parallel vectors never meet.

g) **TRUE** The magnitude of a vector cannot be negative

h) **FALSE** The equation of a plane will always have \( \overrightarrow{0} \) as the initial point.

i) **TRUE** The dot product of two matrices in 3-space will not be a vector in 3-space

j) **FALSE** All vectors in a subspace of \( \mathbb{R}^3 \) will be ordered pairs.
For questions 4-8 below you should match each type of problem with 4 choices that apply to the solving of the problem. Each correct choice will be worth 1 point. Only the first 4 choices will be recorded. There may be more than four correct choices, and some choices can be used more than once. Please record your choices in the blanks at the right.

**CHOICES:**

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<tr>
<th>a) A stochastic Matrix</th>
<th>b) $\bar{x} = P_0 + i\bar{v}$</th>
<th>c) $\bar{u} \cdot \bar{v}$</th>
<th>d) $\bar{x} = P_0 + \text{Span}{\bar{v}_1, \bar{v}_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e) The Zero Vector</td>
<td>f) $\text{det}[A]$</td>
<td>g) $[A \mid \bar{b}]$</td>
<td>h) Parametric solutions</td>
</tr>
<tr>
<td>i) A homogeneous system</td>
<td>j) $\text{col}[A]$</td>
<td>k) $T^n S_0$</td>
<td>m) An Identity Matrix</td>
</tr>
<tr>
<td>n)</td>
<td>o) Position vectors</td>
<td>p) $</td>
<td>\bar{v}</td>
</tr>
</tbody>
</table>

4) (4 points) A Markov Process Problem

5) (4 points) A Linear Combination Problem

6) (4 points) A Geometric Vector Problem

7) (4 points) A Subspace Problem

8) (4 points) A Linear Dependence Problem

\[ \begin{array}{cccccc}
A & K & m & i & e \\
\hline
u & g & h & j & d \\
\hline
b & d & c & p & o \\
\hline
i & d & b & e & h & n & g \\
\hline
g & f & i & e & h & n & g \\
\end{array} \]
9) For all real values of \(t, r,\) and \(s,\) let \(\vec{W} = \{(2t - r, 3t + 2s, 5s)\}, \vec{V} = \{(4t + s, 6 - 8s, -t + 2s)\},\) and \(\vec{X} = \{(2t + 4t, (1s + 3s))\}\) be vector subsets.

a) (2 points) Which of these is not a subspace and why?

\[\vec{V}\] does not have the zero vector.

b) (3 points) Which of these is a subset of \(R^2\) and how do you know?

\[\vec{X}\] vectors from \(R^2\) 2 entries

\[\vec{X} = t(2, 4) + s(1, 3)\]

plane or \(R^2\)

d) (5 points) Write \(\vec{W}\) as a span.

\[\vec{W} = \text{Span}\{(2, 3, 0), (-1, 0, 0), (0, 2, 5)\}\]
10) The chart below shows the weekend work schedule at the local Starbucks. The shop would like to reduce the number of workers to 3, but must maintain the same total number of hours of operation.

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Saturday</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Sunday</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Putting this information in an augmented matrix in reduced echelon form yields:

\[
\begin{bmatrix}
1 & 0 & 0 & -6 & 0 \\
0 & 1 & 0 & 3 & 0 \\
0 & 0 & 1 & 2 & 0
\end{bmatrix}
\]

a) (2 points) Write a parametric equation that shows the dependency among the workers.

\[
\mathbf{0} = 6t \mathbf{w}_1 - 3t \mathbf{w}_2 - 2t \mathbf{w}_3 + t \mathbf{w}_4
\]

b) (4 points) Write a linear combination equation for each employee.

\[
\begin{align*}
\mathbf{w}_1 &= \frac{1}{2} \mathbf{w}_2 + \frac{1}{3} \mathbf{w}_3 - \frac{1}{6} \mathbf{w}_4 \\
\mathbf{w}_2 &= 2 \mathbf{w}_1 - \frac{2}{3} \mathbf{w}_3 + \frac{1}{3} \mathbf{w}_4 \\
\mathbf{w}_3 &= 3 \mathbf{w}_1 - \frac{3}{2} \mathbf{w}_2 + \frac{1}{2} \mathbf{w}_4 \\
\mathbf{w}_4 &= -6 \mathbf{w}_1 + 3 \mathbf{w}_2 + 2 \mathbf{w}_3
\end{align*}
\]

b) (4 points) The store currently meets the operating hour schedule using the following formula:

\[
S = 5w_1 + 4w_2 + 4w_3 + 2w_4
\]

Can they find a schedule that allows them to eliminate one worker if no worker may work more than 10 hours a day? If so, state the new schedule, you need only find one.

\[
S_{\mathbf{w}_1} = 6\frac{1}{2} \mathbf{w}_2 + 5\frac{2}{3} \mathbf{w}_3 + 1\frac{1}{6} \mathbf{w}_4
\]
1. (2 points each) Mark each statement with “True” or “False” (not “T” or “F”)

a)  **TRUE** Vector addition is defined as matrix addition.

b)  **FALSE** The initial state of a Markov process changes the exact stable matrix.

c)  **TRUE** All homogeneous systems are consistent.

d)  **TRUE** Parallel vectors can cross.

e)  **TRUE** All vectors in a subspace of \( \mathbb{R}^3 \) will be ordered triples.

f)  **FALSE** All rows in a Markov analysis matrix add to 1.

g)  **FALSE** The magnitude of a vector can be negative

h)  **TRUE** The span of a line will always have \( \vec{0} \) as the initial point.

i)  **FALSE** The dot product of two matrices in 3-space will be a vector in 3-space

j)  **FALSE** All linear combinations are parametric.
2) Given \( A = \begin{bmatrix} 2 & -4 & -8 \\ 3 & -1 & 3 \\ -5 & 3 & -1 \end{bmatrix} \) \( \xrightarrow{rref} \) \( \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \)

a) (5 points) Write a parametric equation that shows \((12, -2, -2)\) is a linear combination of the column vectors of \( A \).

\( c_1 = -2 - 2t \quad c_2 = -4 - 3t \quad c_3 = t \)

\[ \begin{align*}
1) \quad (12, -2, -2) &= (-2 - 2t)(2, 3, -5) + (-4 - 3t)(-4, -1, 3) \\
& \quad + t(-8, 3, -1)
\end{align*} \]

\[ \begin{align*}
2) \quad \begin{bmatrix} 12 \\ -2 \\ -2 \end{bmatrix} &= (-2 - 2t)\begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} + (-4 - 3t)\begin{bmatrix} -4 \\ -1 \\ 3 \end{bmatrix} + t\begin{bmatrix} -8 \\ 3 \\ -1 \end{bmatrix}
\end{align*} \]

\[ \begin{align*}
3) \quad (12, -2, -2) &= (-2 - 2t)\vec{v}_1 + (-4 - 3t)\vec{v}_2 + t\vec{v}_3 \quad \text{(Labeled A)}
\end{align*} \]

b) (5 points) Are the column vectors of \( A \) linearly dependent? If so, write a parametric dependency equation.

\[ \text{YES} \quad \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

\( c_1 = -2t \quad c_2 = -3t \quad c_3 = t \)

\[ \begin{align*}
0 &= -2t(2, 3, -5) + -3t(-4, -1, 3) + t(-8, 3, 1)
\end{align*} \]

or other forms

\[ t \neq 0 \]
3) A group of students on campus were surveyed about their eating habits and it was found they ate only burgers, pizza and enchiladas. The night the survey was taken 50% had eaten burgers, 30% had had pizza and 20% had eaten enchiladas. Their pattern was, if they ate burgers one night, 35% would again have a burger the next night, 25% would switch to pizza and 40% would switch to enchiladas. Of those who had pizza on any given night, 20% would have a burger the next night, 70% would stick with pizza and 10% would switch to enchiladas. Of those who had enchiladas on a given night, 20% would switch to burgers, 20% would switch to pizza and 60% would stay with enchiladas for the next night.

a) (3 points) Write a transition matrix for this situation. Be sure to clearly label the matrix

\[
T = \begin{bmatrix}
0.35 & 0.20 & 0.40 \\
0.25 & 0.70 & 0.10 \\
0.40 & 0.20 & 0.60
\end{bmatrix}
\]

b) (2 points) Write and label the initial state matrix for this situation.

\[
S_0 = \begin{bmatrix}
0.50 \\
0.30 \\
0.20
\end{bmatrix}
\]

c) (1 point) What is the time period of each state in this situation?

1 Night (Day)

d) (4 points) Write an equation that will compute the percent of students eating each type of food after one week and interpret the solution. (Round your answers to the nearest percent)

\[
S_7 = T^7S_0
\]

\[
S_7 = \begin{bmatrix}
0.35 & 0.20 & 0.40 \\
0.25 & 0.70 & 0.10 \\
0.40 & 0.20 & 0.60
\end{bmatrix}^7 \begin{bmatrix}
0.50 \\
0.30 \\
0.20
\end{bmatrix} = \begin{bmatrix}
0.24 \\
0.42 \\
0.32
\end{bmatrix}
\]

28% will eat burgers, 38% pizza, 35% enchiladas after 1 week.

e) (5 points) If this situation continues indefinitely, there will be an expected ratio of students who will be eating each type of food on any given night. What are those exact ratios? Show your augmented matrix for full credit.

\[
\begin{bmatrix}
-0.65 & 2 & 2 & 0 \\
0.25 & -3 & 2 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

\[
\text{rref} \rightarrow \begin{bmatrix}
1 & 0 & 0 & 0.417 \\
0 & 1 & 0 & 0.365 \\
0 & 0 & 1 & 0.2985
\end{bmatrix}
\]

\[
\frac{4}{17} \text{ will eat burgers, } \frac{36}{85} \text{ pizza, } \frac{29}{85} \text{ enchiladas}
\]
4) The chart below shows the weekend work schedule at the local Starbucks. The shop would like to reduce the number of workers to 3, but must maintain the same total number of hours of operation.

<table>
<thead>
<tr>
<th>Workers</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>6</td>
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Putting this information in an augmented matrix in reduced echelon form yields:

\[
\begin{bmatrix}
1 & 0 & 0 & 2 & 0 \\
0 & 1 & 0 & 3 & 0 \\
0 & 0 & 1 & -6 & 0 \\
\end{bmatrix}
\]

a) (2 points) Write a parametric equation that shows the dependency among the workers.

\[
\begin{bmatrix}
\vec{\theta} = -2 + w_1 - 3t w_2 + 6t w_3 + t w_4 \\
\end{bmatrix}
\]

b) (4 points) Write a linear combination equation for each employee.

\[
\begin{align*}
w_1 &= -\frac{3}{2} w_2 + 3 w_3 + \frac{1}{2} w_4 \\
w_2 &= -\frac{2}{3} w_1 + 2 w_3 + \frac{1}{3} w_4 \\
w_3 &= \frac{1}{3} w_1 + \frac{1}{2} w_2 - \frac{1}{6} w_4 \\
w_4 &= 2 w_1 + 3 w_2 - 6 w_3
\end{align*}
\]

b) (4 points) The store currently meets the operating hour schedule using the following formula:

\[
S = 5w_1 + 4w_2 + 4w_3 + 2w_4
\]

Can they find a schedule that allows them to eliminate one worker if no worker may work more than 10 hours a day? If so, state the new schedule, you need only find one.

\[
\begin{align*}
S_{w_1} &= -3.5 w_2 + 19 w_3 + 7.5 w_4 & \text{NO} \\
S_{w_2} &= 1.5 w_1 + 12 w_3 + 3.5 w_4 & \text{NO} \\
S_{w_3} &= 6.5 w_1 + 6 w_2 + \frac{4}{3} w_4 & \text{YES} \\
\end{align*}
\]
5) For all real values of \( t, r, \) and \( s, \) let, \( \vec{W} = \{(4t+s, 6-8s, -t+2s)\}, \vec{V} = \{(2t,4t),(1s,3s)\} \) and \( \vec{X} = \{(2t-r, 3t+2s, 5s)\} \) be vector subsets.

a) (2 points) Which of these is not a subspace and why?

\[ \vec{W} \text{ does not contain the zero vector} \]

b) (3 points) Which of these is a subset of \( \mathbb{R}^2 \) and how do you know?

\[ \vec{V} \text{ vectors are from } \mathbb{R}^2 \text{ 2 entries} \]

c) (5 points) Write \( \vec{X} \) as a span.

\[ \vec{X} = \text{Span} \left\{ (2,3,0), (-1,0,0), (0,2,5) \right\} \]

d) (5 points) Write the vectors of \( \vec{V} \) as a linear combination equation and describe its geometric interpretation.

\[ \vec{V} = t(2,4) + s(1,3) \]

A plane (or) \( \mathbb{R}^2 \)
For questions 6-10 below you should match each type of problem with 4 choices that apply to the solving of the problem. Each correct choice will be worth 1 point. Only the first 4 choices will be recorded. There may be more than four correct choices, and some choices can be used more than once. Please record your choices in the blanks at the right.

**CHOICES:**

<table>
<thead>
<tr>
<th>a) A stochastic Matrix</th>
<th>b) ( \vec{x} = P_0 + \vec{v} )</th>
<th>c) ( \vec{u} \cdot \vec{v} )</th>
<th>d) ( \vec{x} = P_0 + \text{Span}{\vec{v}_1, \vec{v}_2} )</th>
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<td>j) ( col[A] )</td>
<td>k) ( T^*S_0 )</td>
<td>m) An Identity Matrix</td>
</tr>
<tr>
<td>n) ( \vec{x} = c_1 \begin{bmatrix} v_1 \end{bmatrix} + c_2 \begin{bmatrix} v_1 \end{bmatrix} + \ldots + c_k \begin{bmatrix} v_1 \end{bmatrix} )</td>
<td>o) Position vectors</td>
<td>p) (</td>
<td>\vec{v}</td>
</tr>
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6) (4 points) A Subspace Problem

7) (4 points) A Linear Dependence Problem

8) (4 points) A Markov Process Problem

9) (4 points) A Linear Combination Problem

10) (4 points) A Geometric Vector Problem

\[ \begin{array}{cccccccc}
\text{i} & \text{j} & \text{d} & \text{b} & \text{e} & \text{k} & \text{i} & \text{n} & \text{g} \\
\text{m} & \text{g} & \text{j} & \text{i} & \text{c} & \text{h} & \text{n} & \text{g} \\
\text{a} & \text{k} & \text{m} & \text{i} & \text{e} \\
\text{n} & \text{g} & \text{a} & \text{g} & \text{j} & \text{d} \\
\text{h} & \text{b} & \text{d} & \text{c} & \text{p} & \text{o} \\
\end{array} \]