Circle the lecture you are registered for: X1 (12PM) or F1 (2PM)

1) You should have already checked into the exam and had your calculator approved. No exam will be accepted from a student who does not check in before they start the exam.

2) No hats or dark sunglasses. All hats are to be removed.

3) All book bags should be closed and placed under your seat. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.

4) No cell phones. Turn them off now. If you are seen with a cell phone in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece.

5) No iPods or MP3s players, etc. Same rules as with cell phones.

6) If you have a question, raise your hand and a proctor will come to you. Once you stand up you are done with the exam. If you have to use the bathroom, do so now. You will not be permitted to leave the room and return during the exam.

7) Every exam is worth a total of 50 points. Check to see that you have all of the pages. Including this cover sheet, each exam has 8 pages.

8) Be sure to print your proper name clearly at the top of this page. Also circle the lecture for which you are registered.

9) If you finish early, quietly and respectfully get up and hand in your exam. You will need to show a picture ID when you hand in your exam. No exam will be accepted without ID.

10) When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on. We will come to you to collect the exams. Pull your IDs out now so that you have them ready.

11) To ensure that you receive full credit, show all of your work. Provide all systems of equations and original and final matrices used in solving problems. Indicate all row operations and write detailed conclusions where applicable.

12) You may use a calculator.

13) Good luck. You have 75 minutes to complete this exam.
1a) (3 pts) Define what it means for a system of equations to be inconsistent.

A system of equations is inconsistent if it has no solutions.

1b) (6 pts) Find a value for $k$ that makes the following system of equations inconsistent.

$$
\begin{align*}
  x + 2y + 8z &= -8 \\
  3x + 2y + 12z &= -16 \\
  -x - y + kz &= -5
\end{align*}
$$

$$
\begin{bmatrix}
  1 & 2 & 8 & | & -8 \\
  3 & 2 & 12 & | & -16 \\
  -1 & -1 & k & | & -5
\end{bmatrix}
$$

$$
\begin{align*}
  R_2 &= R_2 - 3R_1 \\
  R_3 &= R_3 + R_1
\end{align*}
$$

$$
\begin{bmatrix}
  1 & 2 & 8 & | & -8 \\
  0 & -4 & -12 & | & 8 \\
  0 & 1 & k+8 & | & -13
\end{bmatrix}
$$

$$
\begin{align*}
  R_2 &= -\frac{1}{4}R_2 \\
  R_3 &= R_3 - R_2
\end{align*}
$$

$$
\begin{bmatrix}
  1 & 2 & 8 & | & -8 \\
  0 & 1 & 3 & | & -2 \\
  0 & 1 & k+8 & | & -13
\end{bmatrix}
$$

$$
\begin{bmatrix}
  1 & 2 & 8 & | & -8 \\
  0 & 1 & 3 & | & -2 \\
  0 & 0 & k+5 & | & -11
\end{bmatrix}
$$

If $k = -5$ then the system is inconsistent.
2a) (2 pts) Give one augmented matrix which can be used to solve the following systems of equations simultaneously.

\[
\begin{align*}
2x + 3y + 7z &= -3 \\
4x - 3y + 8z &= 9 \\
-x - 6y - 7z &= -18
\end{align*}
\quad \text{and} \quad
\begin{align*}
2x + 3y + 7z &= 27 \\
4x - 3y + 8z &= 6 \\
-x - 6y - 7z &= -15
\end{align*}
\]

\[
\begin{bmatrix}
2 & 3 & 7 & -3 & 27 \\
1 & -3 & 8 & 9 & 6 \\
-1 & -6 & -7 & -18 & -15
\end{bmatrix}
\]

2b) (3 pts) Give solutions to each of the systems of equations above.

\[
\begin{bmatrix}
1 & 0 & 0 & -13y & 118 \\
0 & 1 & 0 & -113/3 & 106/3 \\
0 & 0 & 1 & 5y & -415
\end{bmatrix}
\]

**first system** \[ x = -13y \] \[ y = -113/3 \] \[ z = 5y \]

**second system** \[ x = 118 \] \[ y = 106/3 \] \[ z = -415 \]
3) (7 pts) Find all solutions to the system of equations below.

\[
\begin{align*}
3x - 2y - z - 11w &= 4 \\
-x + 4y + 6z + 8w &= 29 \\
-2x - 3y + 5z - 10w &= -7
\end{align*}
\]

\[
\begin{bmatrix}
3 & -2 & -1 & -11 & | & 4 \\
-1 & 4 & 6 & 8 & | & 29 \\
-2 & -3 & 5 & -10 & | & -7
\end{bmatrix}
\]

\[
\text{RREF} \rightarrow 
\begin{bmatrix}
1 & 0 & 0 & -2 & | & 5 \\
0 & 1 & 0 & 3 & | & 4 \\
0 & 0 & 1 & -1 & | & 3
\end{bmatrix}
\]

\[w = t\]

\[x = -2t + 5\]
\[y + 3t = 4\]
\[z - t = 3\]

\[w = t\]

\[x, y, z, w = (5 + 2t, 4 - 3t, 3 + t, t)\]

\[\text{for all } t\]
4) (5 pts) Plot the feasibility region defined by the following system of inequalities. Find the locations of all corner points algebraically and include them on your plot.

(i) \( x + 3y \leq 18 \)
(ii) \( x + y \leq 8 \)
(iii) \( 2x + y \leq 14 \)
(iv) \( x \geq 0, y \geq 0 \)

(i) \( x + 3y = 18 \)
\[ \text{Intersection: } (18, 0), (0, 6) \]
\[ \text{Test: } (0, 0) \implies 0 \leq 18 \checkmark \]

(ii) \( x + y = 8 \)
\[ \text{Intersection: } (8, 0), (0, 8) \]
\[ \text{Test: } (0, 0) \implies 0 \leq 8 \checkmark \]

(iii) \( 2x + y = 14 \)
\[ \text{Intersection: } (7, 0), (0, 14) \]
\[ \text{Test: } (0, 0) \implies 0 \leq 14 \checkmark \]

(i) and (ii)

\[ x + 3y = 18 \]
\[ x + 7 = 8 \]
\[ 2y = 10 \]
\[ y = 5 \]
\[ x = 3 \]

(ii) and (iii)

\[ x + y = 8 \]
\[ 2x + y = 14 \]
\[ -x = -6 \]
\[ x = 6 \]
\[ y = 2 \]
5) (8 pts) The Pennsylvania Power Company runs three power plants which burn coal, oil and gas. Each day the coal plant operates it generates 50,000 kilowatt hours (kWh) at a cost of $12,000. The oil plant generates 35,000 kWh at a cost of $9,500 each day it operates and the natural gas plant generates 45,000 kWh at a cost of $15,000 each day it operates.

The coal plant emits 35 pounds (lbs) of Sulfur and 75 lbs of dust per day of operation. The oil plant emits 25 lbs of Sulfur and 65 pounds of dust each day it operates, and the natural gas plant emits 15 lbs of Sulfur and 50 pounds of dust per day of operation. Due to EPA regulations, the plants can emit a total of at most 400 lbs of Sulfur and 900 lbs of dust per week.

Set up (but do not solve) a linear program that will determine the number of days to run each plant per week in order to generate at least 700,000 kWh at minimum cost.

\[
\begin{align*}
X &= \text{# of days to run coal plant}, \\
Y &= \text{# of days to run oil plant}, \\
Z &= \text{# of days to run gas plant}.
\end{align*}
\]

\[
\begin{align*}
\text{Minimize cost} & \quad C = 12000X + 9500Y + 15000Z \\
\text{Subject to:} & \quad 50000X + 35000Y + 45000Z \geq 700000 \quad \text{(kWh)} \\
& \quad 35X + 25Y + 15Z \leq 400 \quad \text{(Sulfur)} \\
& \quad 75X + 65Y + 50Z \leq 900 \quad \text{(Dust)} \\
& \quad X \geq 0, \quad Y \geq 0, \quad Z \geq 0 \quad \text{(non-negative)} \\
& \quad X \leq 7, \quad Y \leq 7, \quad Z \leq 7 \quad \text{(time constraint)}
\end{align*}
\]
6a) (3 pts) Explain the role of slack variables in setting up the initial simplex table for a linear program.

The use of slack variables converts a system of inequalities into a system of equations that can be represented by an augmented matrix.

6b) (3 pts) Give the initial simplex table for the following linear program. (Do not solve the linear program.)

maximize: \[ z = 2x_1 + 6x_2 - 7x_3 \]
subject to:
\[
\begin{align*}
3x_1 + 7x_2 - 4x_3 & \leq 24 \\
-2x_1 + 7x_2 + 8x_3 & \leq 9 \\
5x_1 - 2x_2 + 3x_3 & \leq 35 \\
x_1 & \geq 0 \\
x_2 & \geq 0 \\
x_3 & \geq 0
\end{align*}
\]

Standard form: \[ z - 2x_1 - 6x_2 + 7x_3 = 0 \]

slack variables:
\[
\begin{align*}
3x_1 + 7x_2 - 4x_3 + x_4 & = 24 \\
-2x_1 + 7x_2 + 8x_3 + x_5 & = 9 \\
5x_1 - 2x_2 + 3x_3 + x_6 & = 35
\end{align*}
\]

\[
\begin{bmatrix}
1 & -2 & -6 & 7 & 0 & 0 & 0 & 0 \\
0 & 3 & 7 & -4 & 1 & 0 & 0 & 24 \\
0 & -2 & 7 & 8 & 0 & 1 & 0 & 9 \\
0 & 5 & -2 & 3 & 0 & 0 & 1 & 35
\end{bmatrix}
\]
7) (10 pts) Charlie's Chocolate Factory makes cupcakes, brownies, and danishes. A batch of cupcakes requires 100 pounds (lbs) of flour and 30 lbs of sugar. A batch of brownies requires 100 lbs of flour and 50 lbs of sugar while a batch of danishes uses 200 lbs of flour and 20 lbs of sugar. Charlie must use 1200 lbs of flour and 200 lbs of sugar immediately so that his supplies do not expire. Determine all possible combinations of batches of cupcakes, brownies and danishes that Charlie can make to exhaust all of his flour and sugar.

\[ X = \text{# of batches of cupcakes} \]
\[ Y = \text{# of batches of brownies} \]
\[ Z = \text{# of batches of danishes} \]

\[ 100X + 100Y + 200Z = 1200 \] (flour)
\[ 30X + 50Y + 20Z = 200 \] (sugar)

\[
\begin{bmatrix}
100 & 100 & 200 & 1200 \\
30 & 50 & 20 & 200
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 4 & 1200 \\
0 & 1 & -2 & -8
\end{bmatrix}
\]

\[ 2 = t \]

\[ x = 20 - 4t \quad 20 - 4t \geq 0 \quad t \leq 5 \]
\[ y = -8 + 2t \quad -8 + 2t \geq 0 \quad t \geq 4 \]
\[ z = t \quad t \geq 0 \quad t \geq 0 \]

The value of \( t \) must be 4 or 5.

Charlie can make 4 batches of cupcakes, and 4 batches of danishes or 2 batches of brownies and 5 batches of danishes.