1) Using the system of linear inequalities below:

\[
\begin{align*}
4x_1 + 10x_2 &\leq 100 \\
2x_1 + x_2 &\leq 22 \\
3x_1 + 3x_2 &\leq 39 \\
x_1 &\geq 0, \quad x_2 &\geq 0
\end{align*}
\]

\((0, 10) \quad (25, 0) \quad (0, 0)\)

\((0, 22) \quad (11, 0) \quad (0, 0)\)

\((0, 13) \quad (13, 0) \quad (0, 0)\)

a) (5 points) Graph the feasibility region.

(Show work that allows the grader to understand the process of your thinking.)

b) (3 points) Find the corner points for the region above.

(Show work that allows the grader to understand the process of your thinking.)

\[
\begin{align*}
4x_1 + 10x_2 &= 100 \\
2x_1 + 5x_2 &= 50 \\
3x_1 + 3x_2 &= 39 \\
x_1 + x_2 &= 13 \\
2x_1 + x_2 &= 22
\end{align*}
\]

\[
\begin{align*}
2(13-x_2) + 5x_2 &= 50 \\
2x_1 - 2x_2 + 5x_2 &= 50 \\
3x_2 &= 24 \\
x_1 &= 3 - x_2
\end{align*}
\]

\[
\begin{align*}
x_1 &= 13 - 8 \\
x_2 &= 8 \\
x_1 &= 9 \\
x_2 &= 4
\end{align*}
\]

(9, 4)

(5, 8)

- \(x_2 = 4\)
- \(x_1 = 9\)
- \(x_1 = 13 - x_2\)

\(x_1 = 9\), \(x_2 = 4\)

\(3x_2 = 24\)

\(2x_1 - 2x_2 + 5x_2 = 50\)

\(2(13-x_2) + 5x_2 = 50\)

\(2x_1 + x_2 = 22\)

\(3x_1 + 3x_2 = 39\)

\(4x_1 + 10x_2 = 100\)

\(2x_1 + x_2 = 22\)

\(x_1 \geq 0, \quad x_2 \geq 0\)

\(x_1 = 13 - x_2\)

\(x_1 = 13 - 8\)

\(x_2 = 8\)

\(x_1 = 9\)

\(x_2 = 4\)

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\(x_1 = 9\)
2) (5 points) Use lines of constancy to evaluate the objective function over the **unbounded region** shown below. State a conclusion using point names only, no coordinates or optimum values are needed. *(Show work that allows the grader to understand the process of your thinking.)*

Objective: Minimize $Z = 3x + 4y$ over the unbounded region shown below.

\[ 4U = Z - 3x \]
\[ U = \frac{Z}{4} - \frac{3}{4}x \]
\[ Z = \frac{Z}{4} - \frac{3}{4}x \]
\[ \begin{array}{c|c}
100 & y = 25 - \frac{3}{4}x \\
400 & y = 100 - \frac{3}{4}x \\
\end{array} \]

**THE MINIMUM IS ACHIEVED AT POINT C**

3) (2 points) Why was it necessary to use lines of constancy to solve the problem above?

**THE REGION IS NOT POLYGONAL**

**IT IS UNBOUNDED.**
4) (5 points) Set up the following application problem, but do not solve.

The Crazy Cookie Company makes three types of cookies, Crumbly, Crunchy, and Chunky. The manager has been asked to maximize the weekly profit at the local branch. The profit on a Crumbly cookie is 30¢, on a Crunchy is 40¢ and on a Chunky is 50¢. The store has 160 man-hours a week to produce cookies. For each type it takes 90 minutes to produce a batch of 100 cookies. They have 200 lbs of flour available each week and the Crumbly takes 1 pound per batch, the Crunchy 2 pounds per batch and the Chunky 3 pounds per batch. They have 250 lbs of sugar on hand each week; the Crumbly takes 1.5 pound per batch, the Crunchy 2 pounds per batch and the Chunky 3 pounds per batch.

**OBJECTIVE:**

Maximize Profit

\[ P = 3x + 4y + 5z \]

**VARIABLES:**

\[ x = \text{# of batches of Crumbly to make} \]
\[ y = \text{# of batches of Crunchy to make} \]
\[ z = \text{# of batches of Chunky to make} \]

**CONSTRAINTS:**

Man-hours

\[ 90x + 90y + 90z \leq 160 \text{ hours} \]

Flour

\[ x + 2y + 3z \leq 200 \text{ pounds} \]

Sugar

\[ 1.5x + 2y + 3z \leq 250 \text{ pounds} \]

5) (5 points) Put the matrix below in row echelon form by hand.

Clearly label your operations from each step to the next.

\[
\begin{bmatrix}
1 & 3 & 4 & 4 \\
-5 & -10 & 0 & 2 \\
-2 & 4 & 30 & a
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 4 & 4 \\
0 & 5 & 20 & 22 \\
0 & 10 & 38 & 8 + a
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 4 & 4 \\
0 & 1 & 4 & 22/5 \\
0 & 10 & 38 & 8 + a
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 4 & 4 \\
0 & 1 & 4 & 22/5 \\
0 & 0 & -2 & -36 + a
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 4 & 4 \\
0 & 1 & 4 & 22/5 \\
0 & 0 & 1 & 18 - 1/2 a
\end{bmatrix}
\]
6) Answer the questions below (2 points each) using these matrices:

\[ A = \begin{bmatrix} 3 & 2 & 0 & 0 \\ c & 6 & 5 & 2 \\ 2c & 12 & 10 & d \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 5 & 2 & 1 \\ 7 & 8 & -2 & 4 \\ 0 & 0 & 0 & c \end{bmatrix} \]

\[ E = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 6 & 4 & 9 \\ 4 & -2 & 0 & a \end{bmatrix}, \quad F = \begin{bmatrix} -5 & 10 & -15 & a \\ 1 & -2 & 3 & b \\ 7 & 8 & 9 & c \end{bmatrix} \]

a) Which matrices are in row echelon form? \( B \& G \)

b) What is the size of matrix C? \( 3 \times 4 \)

c) What is the solution for matrix C? \( (2, 5, 7) \)

d) If A is consistent, what is the value of d? \( 4 \)

e) If matrix D is consistent, what is the geometric shape of the solution? \( \text{PLANE} \)

f) If matrix D is inconsistent, what do you know about the value of c? \( c \neq 0 \)

\( g) \) How does matrix G differ from all the others? \( \text{NOT AUGMENTED} \)

h) What type of system is represented by matrix B? \( \text{MULTI-SYSTEM} \)
7) Set up an augmented matrix to represent the system below and find the solution.

\[
\begin{align*}
    x_1 + 2x_2 - 4x_3 - x_4 &= 7 \\
    2x_1 + 5x_2 - 9x_3 - 4x_4 &= 16 \\
    1x_1 + 5x_2 - 7x_3 - 7x_4 &= 13
\end{align*}
\]

a) (4 points) Augmented matrix and reduced row echelon form:

\[
\begin{bmatrix}
    1 & 2 & -4 & -1 & | & 7 \\
    2 & 5 & -9 & -4 & | & 16 \\
    1 & 5 & -7 & -7 & | & 13
\end{bmatrix}
\xrightarrow{\text{rref}}
\begin{bmatrix}
    1 & 0 & -2 & 3 & | & 3 \\
    0 & 1 & -1 & -2 & | & 2 \\
    0 & 0 & 0 & 0 & | & 0
\end{bmatrix}
\]

b) (3 points) State the solution set:

\[
\begin{align*}
    x_1 &= 3 + 2t - 3s \\
    x_2 &= 2 + t + 2s \\
    x_3 &= t \\
    x_4 &= s
\end{align*}
\]

Solution: \( \{(3 + 2t - 3s, 2 + t + 2s, t, s) \mid t, s \in \mathbb{R} \} \)

\( \text{Solution:} \text{ A Plane} \)

c) (2 points) What is the geometric shape of this solution?

A PLANE

d) (1 point) Where does this solution exist?

\( \mathbb{R}^4 \text{ 4 Space} \)
8) Using the linear program laid out below:

Objective: Minimize \( z = -4x_1 + x_2 \)

Subject to:

\[
\begin{align*}
2x_1 - x_2 & \leq 4 \\
x_1 + x_2 & \leq 5 \\
x_2 & \leq 3 \\
x_1 & \geq 0, \ x_2 & \geq 0
\end{align*}
\]

Over the feasibility Region shown:

(Show work that allows the grader to understand the process of your thinking.)

\[
\begin{align*}
2x_1 - x_2 &= 4 \\
x_1 + x_2 &= 5 \\
x_1 &= 5-x_2 \\
x_1 &= 5-2 = 3 \\
2(5-x_2)-x_2 &= 4 \\
10-2x_2-x_2 &= 4 \\
-3x_2 &= -6 \\
x_2 &= 2
\end{align*}
\]

\[
\begin{array}{c|c|c}
\text{Corner} & z = -4x_1 + x_2 \\
\hline
(0,0) & 0 \\
(0,3) & 3 \\
(2,3) & -5 \\
(3,2) & -10 \star \\
(2,0) & -8 \\
\end{array}
\]

c) (2 points) Conclusion

**The minimum value of**

**+10** **is obtained at**

**The corner (3,2)**
9) (5 points) Graph the first-octant portion of the plane below. 
(Show work that allows the grader to understand the process of your thinking.)

\[10x + 5y + 15z = 30\]

\[\frac{10x}{30} + \frac{5y}{30} + \frac{15z}{30} = 1\]

\[\frac{x}{3} + \frac{y}{6} + \frac{z}{2} = 1\]

10) (5 points) Discuss, using geometric terms, how you know the system in the matrix below is inconsistent.

\[
\begin{bmatrix}
1 & 3 & 2 & 4 \\
2 & 6 & 4 & 9 \\
4 & -2 & 0 & a
\end{bmatrix}
\]

The coefficients of row 2 are each twice those of row 1, however the constant term of row 2 is not twice that of row 1. Therefore the planes are parallel and there is no solution.
11) Short Answer (2 points each).

a) Why must the reduced row echelon form of a homogeneous system always be homogeneous?
BECUSE THE ORIGIN IS A SOLUTION (ZERO SOLUTION)

b) How would you know from the ref form of a system that the solution is unique?
EVERY COLUMN IN THE COEFFICIENT MATRIX CONTAINS A LEADING ONE.

c) Can the solution to a system of two planes be an ordered triple? If so, how? If not, why not?
NO, TWO PLANES CAN NOT INTERSECT IN A SINGLE POINT

d) What must be true if the row rank of an augmented matrix is greater than the number of variables?
THE SYSTEM IS INCONSISTENT

e) If the bottom row of a 2x3 matrix in rref is all zeros, what does that imply geometrically?
THE TWO LINES ARE THE SAME LINE. CO-INCIWENT
Mutz Meals makes three types of doggie treats, Bone, Cheese and Steak. They are made mostly from wheat and soy meal. A batch on Bone treats takes 100 pounds of wheat and 300 pounds of soy meal. A batch of Cheese takes 200 pounds of wheat and 700 pounds of soy meal. A batch of Steak takes 200 pounds of wheat and 500 pounds of soy meal. The company has 1,500 pounds of wheat and 5000 pounds of soy meal on hand. How many batches of each type of treat can they make if they wish to use all the wheat and soy meal on hand?

The system can be set up using the matrix:

\[
\begin{bmatrix}
10 & 200 & 200 \\
300 & 700 & 500 \\
\end{bmatrix}
\begin{bmatrix}
\text{Wheat} \\
\text{Soy Meal} \\
\end{bmatrix}
= \begin{bmatrix}
1500 \\
5000 \\
\end{bmatrix}
lbs
\]

a) Show the rref of this matrix (2 points):

\[
\begin{bmatrix}
1 & 0 & 4 & 1 & 5 \\
0 & 1 & -1 & 1 & 5 \\
\end{bmatrix}
\]

\[
b = 5 - 4t \\
c = 5 + t \\
s = t
\]

b) Write the solution set for this matrix: (4 points)

Solution = \{ (5 - 4t, 5 + t, t) | t \in \mathbb{R} \}

c) Interpret this solution set in terms of the original problem and state the combinations of batchers that should be made. (4 points)

\[
\begin{align*}
0 & \geq 0 \Rightarrow 5 - 4t \geq 0 \Rightarrow t \leq \frac{5}{4} \\
0 & \geq 0 \Rightarrow 5 + t \geq 0 \Rightarrow t \geq -5 \\
0 & \geq 0 \Rightarrow t \geq 0 \\
\end{align*}
\]

0 \leq t \leq 1

They should make 5 batches of Bones, 5 batches of Cheese and no batches of Steak.

\[
or \quad 1 \text{ batch of Bones, 6 batches of Cheese and 1 batch of Steak.}
\]