Math 125 Fall 2016 Exam 1  
Monday, September 26th, 2016

Name: ___________________________ UIN: ____________________

Circle the section where you are picking up the exam:

8am Lecture  2pm Lecture  3pm Lecture

A. CODE THE LETTERS BELOW IN QUESTIONS 91 - 96

B. All bags, purses, jackets, and other personal items should be at the front of the room.

C. If you have a question, raise your hand and a proctor will come to you. If you have to use the bathroom, do so NOW. You will not be permitted to leave the room and return during the exam.

D. No cells phones, i-Pods, MP3 players. Turn them off now. If you are seen these items in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece.

E. Be sure to print your proper name clearly at the top of this page. Also circle the section for which you are registered.

F. If you finish early, quietly and respectfully get up and hand in your exam.

G. When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

H. To ensure that you receive full credit, show all of your work.

I. Good luck. You have 60 minutes to complete this exam.

95. D  
96. C
1. (20 points) A bakery makes angel food and chiffon cupcakes using two common ingredients: sugar and flour. A box of angel food cupcakes requires 3 cups of sugar and 8 cups of flour and sells for $20 per box. A box of chiffon cupcakes requires 3 cups of sugar and 4 cups of flour and sells for $15 a box. The bakery has 57 cups of sugar and 120 cups of flour on a given day. How many boxes of each product should the bakery make to maximize profit?

Solution. -
2. (20 points) Perd’s Puppy Chow makes 3 different kinds of dog food: Chubby Chow (their diet brand), Better than Bacon (their regular brand) and Young Pups (their puppy brand). A bag of Chubby Chow uses 3 lbs of vegetables and 5 lbs of chicken, a bag of Better than Bacon uses 6 lbs of vegetables and 15 lbs of chicken, and a bag of Young Pups uses 18 lbs of vegetables and 20 lbs of chicken. In stock, they have 264 lbs of vegetables and 325 lbs of chicken. How many bags of each type of dog food should they make to use up exactly all their stock?

Solution. -
3. (5 points) Which of these graphs is the feasibility region for the following system of linear inequalities?

\[
\begin{align*}
x + 2y &\geq 6 \\
x + y &\leq 4 \\
x, y &\geq 0
\end{align*}
\]

Graph I

Graph II

Graph III

Graph IV

(A) ★ Graph II.

(B) Graph III.

(C) Graph I.

(D) The correct answer is not here.

(E) Graph IV.
Solution. -
4. (5 points) For the simplex table below, what is the basic feasible solution?

\[
\begin{bmatrix}
1 & 2 & 0 & 5 & 0 & 10 & 0 & 27 \\
0 & 2 & 1 & 0 & 3 & 1 & 1 & 50 \\
0 & 3 & 1 & 1 & 0 & 3 & 0 & 61 \\
0 & 4 & 0 & 1 & 3 & 0 & 17 \\
\end{bmatrix}
\]

(A) \((x_1, x_2, x_3, s_1, s_2, s_3) = (50, 61, 17, 0, 0, 0)\)

(B) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 50, 0, 61, 0, 17)\)

(C) \((x_1, x_2, x_3, s_1, s_2, s_3) = (0, 61, 0, 17, 0, 50)\)

(D) The correct answer is not here.

Solution. -
5. (5 points) Which of the following statements are true?

I. The row rank of \[
\begin{bmatrix}
1 & 1 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
1 & 1 & 4
\end{bmatrix}
\] is 2.

II. The row rank of \[
\begin{bmatrix}
1 & 1 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 4
\end{bmatrix}
\] is 1.

III. The row rank of \[
\begin{bmatrix}
1 & 1 & 4 \\
0 & 1 & 1 \\
0 & 0 & -1
\end{bmatrix}
\] is 3.

(A) ★ All except II are true.

(B) Only I and II are true.

(C) All of the above are true.

(D) Only I is true.

Solution. -
6. (5 points) What is the geometric shape of this solution set? Choose the BEST answer.

\[ \{(2s + t, 2t, 4) \mid t, s \in \mathbb{R}\} \]

(A) The correct answer is not here.
(B) ★ A plane.
(C) A line.
(D) The empty set.
(E) A single point.

Solution.
7. (5 points) Consider the function $z = 4x_1 + x_2$ over the infinite feasibility region drawn below. Which of the following statements are true?

(A) ★ The function can be minimized over the region

(B) The function can be maximized and minimized over the region

(C) The function can be maximized over the region

(D) There is no minimum or maximum of this function over the region

<table>
<thead>
<tr>
<th>Corner Points</th>
<th>$z = 4x_1 + x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = (0,7)</td>
<td>7</td>
</tr>
<tr>
<td>B = (1,5)</td>
<td>9</td>
</tr>
<tr>
<td>C = (4.5, 1.5)</td>
<td>42</td>
</tr>
<tr>
<td>D = (9,0)</td>
<td>36</td>
</tr>
</tbody>
</table>
8. (5 points) Find the value for k that makes the following system of equations INCONSISTENT.

\[
\begin{align*}
    x + 2y + 8z &= -8 \\
    3x + 2y + 12z &= -16 \\
    -x - y + k \cdot z &= -5
\end{align*}
\]

(A) Correct answer is not here.
(B) There is more than one value of \( k \) that makes the system inconsistent.
(C) ★ Only \( k = -5 \) makes the system inconsistent.
(D) Only \( k = -8 \) makes the system inconsistent.

Solution. -
9. (5 points) You are selling atomic, awesome and average cupcakes at your store "Cupcakes! TM". Each atomic cupcake uses 6 oz of sugar, 4 oz of secret ingredient, and 14 oz lucky ingredient and sells for $8. Each awesome cupcake uses 3 oz of sugar, 1 oz of secret ingredient, and 2 oz lucky ingredient and sells for $10. Each average cupcake uses 4 oz of sugar, 2 oz of secret ingredient and 8 oz lucky ingredient and sells for $16. You have 32oz of sugar, 10 oz of secret ingredient and 72 oz of lucky ingredient in stock. To maximize revenue you run the simplex algorithm on the following linear program:

\[
x = \text{# of atomic cupcakes}
\]
\[
y = \text{# of awesome cupcakes}
\]
\[
z = \text{# of average cupcakes}
\]
\[
R = 8x + 10y + 16z \implies R - 8x - 10y - 16z = 0
\]
\[
6x + 3y + 4z \leq 32 \implies 6x + 3y + 4z + s_1 = 32
\]
\[
4x + y + 2z \leq 10 \implies 4x + y + 2z + s_2 = 10
\]
\[
14x + 2y + 8z \leq 72 \implies 14x + 2y + 8z + s_3 = 72
\]

The FINAL simplex table is shown below:

\[
\begin{bmatrix}
1 & 32 & 0 & 4 & 0 & 10 & 0 & 100 \\
0 & -6 & 0 & -2 & 1 & -3 & 0 & 2 \\
0 & 4 & 1 & 2 & 0 & 1 & 0 & 10 \\
0 & 6 & 0 & 4 & 0 & -2 & 1 & 52
\end{bmatrix}
\]

When trying to maximize revenue over this set of contraints, are any ingredients left over?

(A) We have left-over stock for exactly 1 of the ingredients.

(B) The correct answer is not here.

(C) ★ We have left-over stock for exactly 2 of the ingredients.

(D) There are no ingredients left over.

**Solution.**
10. (5 points) Which of the following matrices are in reduced row echelon form?

\[
M = \begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1
\end{bmatrix},
\ N = \begin{bmatrix}
1 & 0 & 3 & 4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 1
\end{bmatrix},
\ P = \begin{bmatrix}
1 & 2 & 0 & 2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(A) Only \( M \)
(B) All of them - \( M, N, \) and \( P \)
(C) ⋄ None of the matrices are in reduced row echelon form
(D) Only \( M \) and \( N \)

**Solution.** -
11. (5 points) Which of the following matrices are in row echelon form?

\[ M = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix} \]

(A) None of the matrices are in row echelon form

(B) ★ Only \( M \)

(C) Only \( M \) and \( N \)

(D) All of them - \( M, N, \) and \( P \)

Solution. -
12. (5 points) The row rank of a matrix might change between REF and RREF.

(A) True
(B) ★ False

Solution.
13. (5 points) The row echelon form of the augmented matrix of a linear system is shown here:

\[
\begin{bmatrix}
1 & 2 & 3 & a \\
0 & 1 & 2 & b \\
0 & 0 & 1 & b \\
\end{bmatrix}
\]

The linear system is in the variable \(x, y\) and \(z\), and \(a\) and \(b\) are real numbers. Express the solution for \(x, y\) and \(z\) in terms of \(a\) and \(b\).

(A) \(\star x = a - b, y = -b, z = b\)

(B) \(x = a, y = b, z = b\)

(C) \(x = a - 9b, y = 3b, z = b\)

(D) \(x = a - 5b, y = 3b, z = b\)

Solution. -
14. (5 points) Consider the system of equations below:

\[
\begin{align*}
4x + 8y + 4z + 4w &= 60 \\
2x + 5y + 2z + 3w &= 36 \\
3x + 7y + 4z + 3w &= 50 \\
x, y, z, w, &\geq 0
\end{align*}
\]

\[
\begin{bmatrix}
4 & 8 & 4 & 4 & 60 \\
2 & 5 & 2 & 3 & 36 \\
3 & 7 & 4 & 3 & 50
\end{bmatrix}
\xrightarrow{RREF}
\begin{bmatrix}
1 & 0 & 0 & 0 & 4 \\
0 & 1 & 0 & 1 & 6 \\
0 & 0 & 1 & -1 & -1
\end{bmatrix}
\]

Assuming you only want to consider non-negative solutions which are integers, how many solutions does this system have?

(A) No Solutions  
(B) 1 Solution  
(C) Infinitely many solutions  
(D) ★ 6 Solutions  
(E) The correct answer is not here

Solution.