1) Given the following Linear Program:

\[
\begin{align*}
\text{PROFIT} &= 8m_1 + 10m_2 + 16m_3 \\
\text{Subject to:} & \quad \begin{cases} 
6m_1 + 3m_2 + 4m_3 \leq 32 \text{ lbs} \\
4m_1 + 1m_2 + 2m_3 \leq 10 \text{ yds} \\
14m_1 + 2m_2 + 8m_3 \leq 72 \text{ days} \\
\end{cases} \\
& \quad m_1, m_2, m_3 \geq 0
\end{align*}
\]

a) (3 pts) Place the system in a Simplex table to determine maximum profit.

\[
\begin{bmatrix}
1 & -8 & -10 & -16 & 0 & 0 & 0 & 0 \\
0 & 6 & 3 & 4 & 1 & 0 & 0 & 32 \\
0 & 4 & 1 & 2 & 0 & 1 & 0 & 10 \\
0 & 14 & 2 & 8 & 0 & 0 & 1 & 72 \\
\end{bmatrix}
\]

\[
\frac{32}{4} = 8
\]

\[
\frac{40}{8} = 5 \checkmark
\]

\[
\frac{72}{8} = 9
\]

b) (3 pts) Circle your first pivot point on the table above and show the table that results from the pivot below.

\[
\begin{bmatrix}
1 & 24 & -2 & 0 & 0 & 8 & 0 & 80 \\
0 & -2 & 1 & 0 & 1 & -2 & 0 & 12 \\
0 & 2 & \frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & 5 \\
0 & -2 & -2 & 0 & 0 & -4 & 1 & 32 \\
\end{bmatrix}
\]

\[
\frac{12}{1} = 12
\]

\[
\frac{5 \cdot \frac{2}{7} = 10 \checkmark
\]

c) (2 pts) State the Basic Feasible Solution for the table from part b above.

\[
\max = 80 \quad @ \quad (0, 0, 5, 12, 0, 32)
\]

d) (2 pts) Choose the correct final solution for this linear program from the choices below.

A) \( Z = 100 \ @ \ (0, 10, 0, 2, 0, 52) \)

B) \( Z = 100 \ @ \ (0, 10, 0) \)

C) \( Z = 80 \ @ \ (0, 12, 1, 0, 0, 56) \)

D) \( Z = 80 \ @ \ (0, 12, 1) \)

e) (3 pts) Indicate any slack that remains in the final basic feasible solution of the table.

\[
\text{There was 2 lbs unused and 52 days unused.}
\]
2) Each True or False question below is worth 3 points. Circle either T or F for you answer. If the statement is false, rewrite it as a true statement.

a) \([A^{-1}B^{-1} = (AB)^{-1}\)  
\(\text{Circle Answer: T } \text{ F} \)  
\(\text{If false, write corrected statement here: } \beta^{-1}A^{-1} = (AB)^{-1} \)  
\(\text{or } A^{-1}\beta^{-1} = (BA)^{-1} \)

b) \((A^{-1})^{-1} = (A^t)^t\)  
\(\text{Circle Answer: T } \text{ F} \)  
\(\text{If only if }\) square

c) If \(A = \begin{bmatrix} 2 & -4 & 6 \\ 3 & -5 & 8 \\ -2 & 6 & -10 \end{bmatrix}\) then \(AA^{-1} = I_3\)  
\(\text{Circle Answer: T } \text{ F} \)

d) Only a square matrix can have a transpose.  
\(\text{Circle Answer: T } \text{ F} \)  
\(\text{All have ...} \)

e) Matrix multiplication is always commutative.  
\(\text{Circle Answer: T } \text{ F} \)  
\(\text{Square ... always -2} \)  
\(\text{comm -> asso - some} \)  
\(\text{not always = never -2} \)  
\(\text{not -1} \)
3) Under what conditions would the statements below always be true? (2 points each)

   Note: "iff" means "if and only if"

   a) \[ A][A] = I \text{ iff} \]
      \[ A \text{ is the identity} \]
      \[ A \text{ is its own inverse} \]

   b) \[ [A][B] = [A][C] \text{ implies } [B] = [C] \text{ iff} \]
      \[ A \text{ is invertible} \]
      \[ A \text{ has an inverse} \]

   c) \[ [A][B] \text{ is defined iff} \]
      \[ \text{The # of columns in } A \text{ is the same as the # of rows in } B \]

4) List three things that must be true about a linear program if you want to use a Simplex Table:

   a) (2 pts) \[ \text{You are looking for a maximum} \]

   b) (2 pts) \[ \text{All constraints are } \leq \text{ a positive number} \]

   c) (2 pts) \[ \text{All variables are } \geq 0. \]
5) Matrix \( S \) below shows the number of certain TVs sold during December at the Best Buy stores in Champaign (C), Danville (D) and Peoria (P). Matrix \( R \) shows the number of items returned in January at each store.

\[
S = \begin{bmatrix}
40 & 30 & 38 & 32'' \\
20 & 42 & 30 & 37'' \\
42 & 54 & 40 & 45'' \\
\end{bmatrix} \quad \text{# of items sold}
\]

\[
R = \begin{bmatrix}
5 & 12 & 8 & \text{Champaign} \\
6 & 7 & 3 & \text{Danville} \\
4 & 10 & 4 & \text{Peoria} \\
\end{bmatrix} \quad \text{# of items returned}
\]

a) (5pts) If the 32'' sold for $650, the 37'' for $875, and the 45'' for $1200, find a matrix expressing the total retail amount sold at each store during December. For full credit, show your matrix operation and clearly label your final matrix.

\[
RP = \begin{bmatrix}
650 \\
875 \\
1200 \\
\end{bmatrix}
\]

\[
\text{RETAIL AMOUNT} = S^t \cdot RP = \begin{bmatrix}
93900 \\
121050 \\
98950 \\
\end{bmatrix}
\]

b) (5pts) After the returns were made in January, (assuming all customers were refunded their full amount), find a matrix expressing how much in retail sales each store actually make during the month of December? Clearly label your work and show your matrix operation for full credit.

\[
\begin{bmatrix}
35 & 20 & 34 & \text{32''} \\
8 & 35 & 20 & \text{37''} \\
34 & 54 & 36 & \text{45''} \\
\end{bmatrix}
\]

\[
\text{ACTUAL RETAIL VALUE SOLD IN DECEMBER OVER}
\]

\[
\begin{bmatrix}
70550 \\
107425 \\
82800 \\
\end{bmatrix}
\]

(look #) \( \cdot \) (size, \( \theta \))

\[
AS^t \cdot RP = \begin{bmatrix}
\text{ Potential } \\
\text{ Value } \\
\text{ Sold } \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
17512.50 \\
10218.75 \\
12112.50 \\
\end{bmatrix}
\]

(look size) \( \cdot \) (size, \( \theta \))

\[
\frac{1}{3}
\]

c) (3 pts) If they sold the returned TV's at 25% off the original price, how much money could each store recover? Clearly label your work and show your matrix operation for full credit.

\[
R \cdot (0.75 \cdot RP) = \begin{bmatrix}
\text{AMOUNT RECOVERED} \\
\text{(look size)} \cdot \text{(size, } \theta) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
17512.50 \\
10218.75 \\
12112.50 \\
\end{bmatrix}
\]
OR \[ R \cdot RP = \text{VALUE RETURNED} = \begin{bmatrix} \$23350 \\ \$13625 \\ \$16150 \end{bmatrix} \]

And \[ \text{RETAIL AMOUNT - VALUE RETURNED} \]

= \[ \text{ACTUAL RETAIL SOLD IN DECEMBER} = \begin{bmatrix} \$70550 \\ \$167425 \\ \$82800 \end{bmatrix} \]
For questions 6—9 below use the following matrices:

\[ A = \begin{bmatrix} 5 & a \\ -a & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & x & -1 \\ 5 & y & 6 \end{bmatrix}, \quad C = \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -2 & 4 \\ 3 & 0 & 6 \end{bmatrix}, \quad E = \begin{bmatrix} -1 & 4 \\ 3 & -1 \\ 0 & 5 \end{bmatrix} \]

6) (3 pt) Find \(2B^t + E\) if possible, if not possible state why.

\[
2 \begin{bmatrix} 2 & 5 \\ x & 4 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 3 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 1x1 \\ 2x+3 & 2y-1 \\ -2 & 17 \end{bmatrix}
\]

7) (3 pt) Find \(A \times E^t\) if possible, if not possible state why.

\[
\begin{bmatrix} 5 & a \\ -a & -5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & 0 \\ 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} -5+4a & 15-a & 5a \\ a-20 & -3a+5 & -25 \end{bmatrix}
\]

8) (3 pt) Solve \(D \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = C\) if possible, if not possible state why.

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = D^{-1} C = \begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}
\]

9) (3 pt) Find \(B \times E \times D^{-1}\) if possible, if not possible state why.

\[
\begin{bmatrix} 1 \\ 2x3 \\ 3x2 \end{bmatrix} \text{ NOT POSSIBLE } \quad \frac{\# \text{ of Columns}}{\# \text{ of Rows}}
\]
For questions 10 - 15 use the Simplex Tables below:

\[
A = \begin{bmatrix}
1 & -4 & 0 & 8 & 0 & 45 \\
0 & -3 & 0 & 5 & 1 & 20 \\
0 & -2 & 1 & 3 & 0 & 9
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1 & -5 & 0 & 0 & 2 & 0 & 0 & 25 \\
0 & 6 & 0 & 1 & 0 & 3 & 6 & 18 \\
0 & 2 & 1 & 0 & 6 & 0 & 1 & 2 \\
0 & -2 & 0 & 0 & -1 & 5 & 2 & 7
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & -2 & 4 & -3 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 6 & 1 & 0 & 0 & 9 \\
0 & -1 & 5 & 7 & 0 & 1 & 0 & 7 \\
0 & 2 & 1 & -1 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
1 & 0 & -5 & -1 & 4 & 98 \\
0 & 0 & 2 & 8 & 0 & 1 \\
0 & 1 & 3 & -4 & -6 & 6
\end{bmatrix}
\]

\[
E = \begin{bmatrix}
1 & 3 & -5 & 0 & 0 & 0 \\
0 & -4 & -1 & 1 & 0 & 7 \\
0 & 1 & 4 & 0 & 0 & 8
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
1 & 3 & 1 & 3 & 0 & 27 \\
0 & 7 & 0 & -2 & 1 & 17 \\
0 & 2 & 1 & -4 & 0 & 8
\end{bmatrix}
\]

10) (3 pt) Which Simplex Table above will not produce a maximum and how do you know?

A HAS A PIVOT COLUMN BUT NO POSITIVE PIVOT VALUE

11) (3 pt) Which table is not produced from a system that meets the criteria for the Simplex Algorithm and how do you know?

E, THERE IS NO SLACK VARIABLE FOR CONSTRAINT # 2

12) (2pt) Which table should be pivoted on the entry whose value is 2?

D OR B

13) (2pt) Which could represent an Initial Simplex Table?

C

14) (2pt) What is the Basic Feasible Solution for Table B?

\((0, 2, 18, 0, 0, 0)\)

15) (3 pt) Which could represent a Final Simplex Table and how do you know?

F THERE ARE NO NEGATIVE ENTRIES IN THE TOP ROW
16) Below is the initial simplex table for a linear program with objective $Z$ with variables $x$ and $y$.

\[
\begin{array}{ccc|ccc|c}
1 & -60 & -50 & 0 & 0 & 0 & 0 \\
0 & 4 & 10 & 1 & 0 & 0 & 100 \\
0 & \hat{2} & 1 & 0 & 1 & 0 & 22 \\
0 & 3 & 3 & 0 & 0 & 1 & 39
\end{array}
\]

10\% = 25
22\% = 11
39\% = 13

a) (3 pts) Circle the pivot point for this table and give its location here $(3, 2)$.

b) (5 pts) State the objective function for the linear program and show the constraint inequalities.

Objective: $Z = 60x + 50y$

Constraints:

\[
\begin{align*}
4x + 10y &\leq 100 \\
2x + y &\leq 22 \\
3x + 3y &\leq 39 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

b) Below are the simplex tables for the remainder of the algorithm. Indicate the basic feasible solution (BFS) for each. (2 points each)

\[
\begin{array}{ccc|ccc|c}
1 & 0 & -20 & 0 & 30 & 0 & 660 \\
0 & 0 & 8 & 1 & -2 & 0 & 56 \\
0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 11 \\
0 & 0 & \frac{3}{2} & 0 & \frac{3}{2} & 1 & 6
\end{array}
\quad \text{and then} \quad
\begin{array}{ccc|ccc|c}
1 & 0 & 0 & 10 & \frac{40}{3} & 0 & 740 \\
0 & 0 & 0 & 1 & 6 & -\frac{16}{3} & 24 \\
0 & 1 & 0 & 0 & 1 & -\frac{1}{3} & 9 \\
0 & 0 & 1 & 0 & -1 & \frac{2}{3} & 4
\end{array}
\]

BFS = $(11, 0, 5, 0, 6)$

\[
\text{Max } 660
\]

c) (3 pts) State the conclusion to the linear program.

$Z$ is maximized @ $(9, 4)$ with

A value of 740
17) Below is the graph of the feasible region for the linear program from question #16.

a) (3 pts) Trace the path the simplex algorithm took to arrive at the solution.

b) (2 pts) Label each corner on the path and indicate the associated maximum value of Z.