Part II - Multiple Choice

Instructions for Entering Multiple Choice Answers: On this exam you will be entering your answers to the multiple choice questions on a scantron sheet that is included with your exam. Always use a Number 2 pencil, maintaining to shade darkly within the bubble. Do NOT cross out your mistakes, but rather erase them thoroughly before entering another answer. Before beginning, please code in your name, UIN, and netid in the appropriate places. In addition, make sure that the version of the exam you are taking, either V1 or V2, is written on one of the blank lines of the scantron sheet. On your exam paper circle the choices you made on the scantron sheet.

1. (5 points) The vectors \( \langle k, 2, 4 \rangle \) and \( \langle k, 5k, 4 \rangle \) are orthogonal only for the following value of \( k \):
   A \( k = 2 \)
   B \( k = -8 \)
   C \( k = -2 \) and \( k = 8 \)
   D No value of \( k \)

2. (5 points) Which of the following is NOT the equation of the line through the points \( (2, 2, 1, 3) \) and \( (-2, 4, 7, 1) \)?
   A \( \vec{x} = \langle 2, 2, 1, 3 \rangle + t\langle 4, -2, -6, 2 \rangle \)
   B \( \vec{x} = \langle -2, 4, 7, 1 \rangle + t\langle -4, 2, 6, -2 \rangle \)
   C \( \vec{x} = \langle 2, 2, 1, 3 \rangle + t\langle 2, -1, -3, 1 \rangle \)
   D \( \vec{x} = \langle -2, 4, 7, 1 \rangle + t\langle -4, 2, 6, -4 \rangle \)

3. (5 points) Consider the following statements.

   A slack variable
   I: appears in a parametric solution and can take on any real value
   II: is a left over variable
   III: measures the degree to which an inequality is satisfied as an equality
   IV: can take on a negative value

Which of these statements is true?
   A All of the statements
   B Only I and III
   C Only III
   D Only III and IV
4. (5 points) Consider the linear programs

maximize \( 2x + 3y + 4z \)
subject to
\[ \begin{align*}
I: & \quad x + y \leq 4 \\
& \quad y + z \leq 5 \\
& \quad z \leq 7 \\
& \quad x \geq 0, y \geq 0, z \geq 0
\end{align*} \]

minimize \( 4x - 3y \)
subject to
\[ \begin{align*}
II: & \quad x + y \leq 4 \\
& \quad 2x - 3y \leq 5 \\
& \quad 4x + 2y \leq 10 \\
& \quad x \geq 0, y \geq 0
\end{align*} \]

maximize \( 5x + 4y - 3z \)
subject to
\[ \begin{align*}
III: & \quad 3x + y + z \leq 7 \\
& \quad 2x + y + 2z \leq 4 \\
& \quad x + y + z \leq 4 \\
& \quad y \geq 0
\end{align*} \]

minimize \( 4x - 3y \)
subject to
\[ \begin{align*}
IV: & \quad x + y \leq 4 \\
& \quad 2x - 3y \leq 5 \\
& \quad 4x + 2y \leq 10 \\
& \quad x \geq 0, y \geq 0
\end{align*} \]

Which of these are solvable by the simplex method as presented in class?
A  I and IV
B  I, III and IV
C  Only IV
D  All of them

5. (5 points) For a linear system with augmented matrix \([A \mid \vec{b}]\) representing three equations in three unknowns, consider the following statements:

I: The row ranks of the coefficient matrix
and augmented matrix are equal

II: \( \vec{b} \) is a linear combination of the columns of \( A \)

III: The RREF of the coefficient matrix is
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Which of these statements are equivalent to the statement that the system is consistent?
A  I
B  II
C  I and II
D  All three

6. (5 points) When solving the system of equations shown below we create an augmented matrix and place it into RREF:

\[
\begin{align*}
x + 2y + 3z &= 4 \\
2x + 3y + 4z &= 5 \\
3x + 4y + 5z &= 6
\end{align*}
\]

The resulting parametric solution
\[
\{(t - 2, -2t + 3, t) \mid t \in \mathbb{R}\}
\]

has 1 parameter, and thus the points in the solution set lie on a line. What is the vector equation of this line?
A  \( \vec{x} = (-2, 3, 0) + t(1, -2, 1) \)
B  \( \vec{x} = (1, -2, 1) + t(-2, 3, 0) \)
C  \( \vec{x} = (-2, 3, 0) + t(-1, -2, -1) \)
D  None of the above
7. (5 points) Let
\[ v = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

Then \( v \) is a linear combination of
A \( u_1 \) and \( u_2 \)
B \( u_1 \) and \( u_3 \)
C \( u_2 \) and \( u_3 \)
D None of the above

8. (2 points) Which of the following represent a possible initial simplex table?

\[ A = \begin{bmatrix} 1 & -2 & -1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 1 & 0 & 6 \\ 0 & 5 & 4 & 0 & 1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 6 \\ 0 & 5 & 4 & 0 & 1 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 & -1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 6 \\ 0 & 5 & 4 & 0 & 1 & 9 \end{bmatrix} \]
A All of the above
B Just matrix \( A \)
C Just matrix \( C \)
D Both matrices \( B \) and \( C \)

9. (2 points) Which entries in the matrix \( A = [a_{ij}] \) below have already been pivoted? The location of the entries is listed by \( a_{ij} \) where \( i = \) row number and \( j = \) column number.

\[ A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \]
A The entry \( a_{31} \)
B The entry \( a_{33} \)
C The entries \( a_{11} \) and \( a_{31} \)
D The entries \( a_{11}, a_{31} \) and \( a_{33} \)

For the following 3 questions, consider the simplex tables below:

\[ A = \begin{bmatrix} 1 & -4 & 0 & 8 & 0 & 45 \\ 0 & -3 & 0 & 5 & 1 & 20 \\ 0 & -2 & 1 & 3 & 0 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -5 & 0 & 0 & 0 \\ 0 & -4 & 1 & 1 & 0 & 7 \\ 0 & 1 & 4 & 0 & 0 & 8 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 & 0 & 3 & 0 & 27 \\ 0 & 7 & 0 & -2 & 1 & 17 \\ 0 & 2 & 1 & -4 & 0 & 8 \end{bmatrix} \]

10. (2 points) Which of the simplex tables above could represent a final simplex table?
A Table \( A \)
B Table \( B \)
C Table \( C \)
D None of these tables represent a final simplex table
11. (2 points) Which table is NOT produced from a system that meets the criteria for the Simplex Algorithm?

- A  Table A
- B  Table B
- C  Table C
- D  All of the above

12. (2 points) Which simplex table will not produce a maximum?

- A  Table A will not produce a maximum
- B  Table B will not produce a maximum
- C  Table C will not produce a maximum
- D  All of the tables will produce a maximum

Consider the following information in answering the subsequent True/False questions.

\[ \vec{u} = (2, 1), \vec{v} = (-4, -2), \vec{w} = (-2, -4) \]

13. (1 point) The vectors \( \vec{u} \) and \( \vec{v} \) are parallel vectors.

- A  True
- B  False

14. (1 point) The slope of the line which vector \( \vec{w} \) lies on is \( \frac{1}{2} \)

- A  True
- B  False

15. (1 point) The vector \( \vec{u} \) must be drawn in the first quadrant.

- A  True
- B  False

16. (1 point) The vector \( \vec{u} \) can be drawn in either \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \).

- A  True
- B  False

17. (1 point) The length of vector \( \vec{w} \) is \( 2\sqrt{5} \)

- A  True
- B  False