Circle your lecture

- B1, MWF 9:00-9:50, Dr. Jason Anema
- X1, MWF 12:00-12:50, Dr. Jason Anema

- You should have already checked into the exam and had your calculator approved. No exam will be accepted from a student who does not check in before they start the exam.
- No hats or dark sunglasses. All hats are to be removed.
- All book bags should be closed and placed under your seat. Do not reach into your bag for anything during the exam. If you need extra pencils or pens, pull them out NOW.
- No cell phones or other electronic devices. Turn them off now. If you are seen with a cell phone in hand during the exam it will be considered cheating, and you will be asked to leave. This includes using it as a time-piece.
- No questions will be permitted during the exam. If you are uncertain about something, put a small note about it on your exam paper and then answer the question as best you can with the information that you are given.
- If you finish early, quietly and respectfully get up, hand in your exam, and show your photo ID.
- When time is up, you will be instructed to put down your writing utensil, and close your exam. Any one who chooses to not close their exam immediately, will have their exam marked and lose all potential points on the page they are writing on.
- To ensure that you receive full credit, show all of your work on the free response section of the exam. Provide all systems of equations and original and final matrices used in solving problems. Indicate all row operations if asked to do so and write detailed conclusions where applicable. Showing your work is NOT required for the multiple choice section.
- You may use a calculator in any calculation that does not explicitly say it must be done by hand.
- At the end of the exam, quickly turn in your test to a proctor and show your Photo ID.
- Good luck. You have 60 minutes to complete this exam.

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

Calculator approved? YES, NO
PART I: Free Response (SHOW ALL OF YOUR WORK)

1. (1 point) Go to the front page of your exam and circle your lecture section.

2. (19 points, 7/3/3/2/2/2) In this problem, we will solve the linear program:

maximize: \[ w = -3x + 6y - 4z \]
subject to:
\[ 10x + 4y - 2z \leq 20 \]
\[ 12x + 6y - 6z \leq 18 \]
\[ -2x + 2y + z \leq 24 \]
\[ x \geq 0, y \geq 0, z \geq 0, \]

using the Simplex Algorithm. (You may use the Pivot Program on your calculator, or perform pivoting on the scratch paper provided)

(a) (7 points) Construct the initial simplex table corresponding to the linear program above. Clearly indicate the position of the pivot point to be used in the Simplex Algorithm.

\[
\begin{array}{cccccc}
1 & 3 & -6 & 4 & 0 & 0 \\
0 & 10 & 4 & -2 & 1 & 0 & 20 \\
0 & 12 & -6 & 0 & 1 & 0 & 24 \\
0 & -2 & 2 & 1 & 0 & 0 & 0 \\
\end{array}
\]

(b) (3 points) Write down the intermediate simplex table you get after performing the first pivot. Clearly indicate the position of the next pivot point to be used in the Simplex Algorithm.

\[
\begin{array}{cccccc}
1 & 15 & 0 & -2 & 6 & 1 & 0 & 18 \\
0 & 2 & 0 & 2 & 1 & -2/3 & 0 & 6 \\
0 & 2 & 1 & -1 & 0 & 1/6 & 0 & 3 \\
0 & -6 & 0 & 3 & 0 & -1/3 & 1 & 18 \\
\end{array}
\]

(c) (3 points) Write down the resulting final simplex table.

\[
\begin{array}{cccccc}
1 & 17 & 0 & 0 & 1 & 1/3 & 0 & 26 \\
0 & 1 & 6 & 1 & 1/2 & -1/3 & 0 & 4 \\
0 & 3 & 1 & 0 & 1/2 & -1/6 & 0 & 7 \\
0 & -9 & 0 & 0 & -3/2 & 2/3 & 1 & 6 \\
\end{array}
\]

(d) (2 points) What is the basic feasible solution of the final simplex table?

\[ (0, 7, 4, 0, 0, 6) \]

(e) (2 points) What is the maximum \( w \)-value in this linear program?

\[ 26 \]

(f) (2 points) At what point in the feasibility region does the maximum occur?

\[ (0, 7, 4) \]
3. (14 points, 7 points each) The Argo Boating Company has five factories around the world, at Athens, Berlin, Chicago, Delhi, and Ithaca. Every day, in an eight hour shift, these factories produce boats according to the table.

<table>
<thead>
<tr>
<th>city/boats</th>
<th>rowboats</th>
<th>sailboats</th>
<th>speedboats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens</td>
<td>20</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Berlin</td>
<td>10</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Chicago</td>
<td>60</td>
<td>270</td>
<td>210</td>
</tr>
<tr>
<td>Delhi</td>
<td>45</td>
<td>210</td>
<td>165</td>
</tr>
<tr>
<td>Ithaca</td>
<td>6</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) The company would like to eliminate the Chicago factory by replacing its production at just the Athens and Berlin factories. Can it do this? If so, indicate the new productions schedules. If not, why not? Explain your reasoning and state a proper conclusion.

\[ \overline{c} = c_1 \overline{A} + c_2 \overline{B} \]

Solve
\[
\begin{bmatrix}
20 & 10 & 60 \\
100 & 40 & 270 \\
80 & 30 & 210
\end{bmatrix}
\rightarrow
\begin{bmatrix}
10 & 1.5 \\
0 & 1.5 \\
0 & 0
\end{bmatrix}
\]

\[ c_1 = 1.5, c_2 = 3 \]

To replace the Chicago factory's production with Athens & Berlin would require operating Athens 12 hours overtime per day and Berlin 24 hours overtime per day, which is not possible.

(b) Alternatively the company would like to eliminate the Delhi factory by replacing its production at just the Athens and Berlin factories. Can it do this? If so, indicate the new productions schedules. If not, why not? Again, explain your reasoning and state a proper conclusion.

\[ \overline{d} = c_1 \overline{A} + c_2 \overline{B} \]

Solve
\[
\begin{bmatrix}
20 & 10 & 45 \\
100 & 40 & 210 \\
80 & 30 & 165
\end{bmatrix}
\rightarrow
\begin{bmatrix}
10 & 1.5 \\
0 & 1.5 \\
0 & 0
\end{bmatrix}
\]

To replace Delhi's production with Athens & Berlin they would run both Athens and Berlin 12 hours overtime per day, hence Athens & Berlin would need to operate 20 hours/day.
4. (10 points, 2 points each) Find the indicated matrices below, or explain why they do not exist. Do not use your calculator, and show all work.

\[
A = \begin{bmatrix} 3 & -2 & 4 \\ 0 & -1 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 5 \\ 7 & 12 \end{bmatrix} \quad D = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad E = \begin{bmatrix} 7 \\ 7 \end{bmatrix} \quad F = \begin{bmatrix} 3 \\ 2 \\ -6 \end{bmatrix}
\]

(a) \(CE\)

\[
\begin{bmatrix} 4 & 5 \\ 7 & 12 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \end{bmatrix} = \begin{bmatrix} 28 + 35 \\ 49 + 84 \end{bmatrix} = \begin{bmatrix} 63 \\ 133 \end{bmatrix}
\]

(b) \(A^T\)

\[
\begin{bmatrix} -3 & 0 \\ -2 & -1 \\ 4 & 7 \end{bmatrix}
\]

(c) \(AF\)

\[
\begin{bmatrix} 3 & -2 & 4 \\ 0 & -1 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 9 - 4 - 24 \\ 0 - 2 - 42 \end{bmatrix} = \begin{bmatrix} -19 \\ -44 \end{bmatrix}
\]

(d) \(C^{-1}\)

\[
\frac{1}{-35} \begin{bmatrix} 12 & -5 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{-12}{35} \quad \frac{5}{35} \\ \frac{-7}{35} \quad \frac{4}{35} \end{bmatrix}
\]

(e) \(BD\)

\[
\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad \text{dimensions don't match, so we can't multiply.}
\]

\[
B \quad \text{and} \quad D
\]
Part II: Multiple Choice (CIRCLE YOUR ANSWERS)  
4 points each

5. The vectors \((k, 1, 1)\) and \((k, -k, 6)\) are orthogonal only for the following value of \(k\):
   (a) \(k = 3\)
   (b) \(k = -2\)
   (c) \(k = -2\) and \(k = 3\)
   (d) No value of \(k\)  

6. Let \(\vec{w} = (-2, -4)\). The slope of the line which vector \(\vec{w}\) lies on is:
   (a) \(\frac{1}{2}\)
   (b) \(-\frac{1}{2}\)
   (c) \(2\)
   (d) \(-2\)
   (e) It is impossible to tell without more information.

7. Consider the following linear program.

\[
\begin{align*}
\text{maximize:} & \quad 2x + 3y - 5z \\
\text{subject to:} & \quad x + y \leq 4 \\
& \quad y + z \leq 5 \\
& \quad z \leq 7 \\
& \quad 3y + 11z \leq 23 \\
& \quad x \geq 0, y \geq 0, z \geq 0,
\end{align*}
\]

To solve this linear program using the simplex algorithm, which of the following statements is true?

(a) Requires no slack variables.
(b) Requires the use of exactly two slack variables.
(c) Requires the use of exactly three slack variables.
(d) Requires the use of exactly four slack variables.
8. When solving the system of equations shown below we create an augmented matrix and place it into RREF:

\[
\begin{align*}
    x + 2y + 3z &= 4 \\
    3x + 4y + 4z &= 5 \\
    2x + 4y + 6z &= 8
\end{align*}
\]

\[
\begin{bmatrix}
    1 & 2 & 3 & 4 \\
    3 & 4 & 4 & 5 \\
    2 & 4 & 6 & 8
\end{bmatrix}
\xrightarrow{RREF}
\begin{bmatrix}
    1 & 0 & -2 & -3 \\
    0 & 1 & 2.5 & 3.5 \\
    0 & 0 & 0 & 0
\end{bmatrix}
\]

The resulting parametric solution has 1 parameter, and thus the points in the solution set lie on a line.

\[
\{(2t - 3, 3.5 - 2.5t, t) | t \in \mathbb{R}\}
\]

What is the vector equation of this line?

(a) \( \vec{x} = (-3, 3.5, 0) + t(2, -2.5, 1) \)  
(b) \( \vec{x} = (-3, 3.5, 0) + t(2, -2.5, 1) \)
(c) \( \vec{x} = (2, -2.5, 0) + t(-3, 3.5, 1) \)
(d) Correct Answer is not here

9. For the linear system

\[
\begin{align*}
    x + 2y + 3z &= 5 \\
    2x - 4y + 5z &= 6 \\
    x + y + z &= 1
\end{align*}
\]

Consider the following statements.

I: The coefficient matrix is invertible
II: \( \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} \) is a linear combination of \( \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \) and \( \begin{bmatrix} 3 \\ 1 \end{bmatrix} \)
III: The coefficient matrix is of full rank
IV: The system is inconsistent.

Which statements are true?

(a) Only I
(b) Only I and II
(c) Only I, II, and III
(d) All of the statements are true.
10. Let \( \vec{w} = (-2, -4) \). The slope of the line which vector \( \vec{w} \) lies on is:

(a) \( \frac{1}{2} \)
(b) \( -\frac{1}{2} \)
(c) \( 2 \)
(d) \( -2 \)
(e) It is impossible to tell without more information.

11. Let \( \vec{u}_1 = (1, 1, 1), \vec{u}_2 = (2, 3, 2), \) and \( \vec{u}_3 = (2, 2, 3) \). The vector \( \vec{v} = (1, 0, 2) \), can be written as a linear combination of \( \vec{u}_1, \vec{u}_2, \) and \( \vec{u}_3, \) as

\[ \vec{v} = c_1 \vec{u}_1 + c_2 \vec{u}_2 + c_3 \vec{u}_3 \]

For which values of constants \( c_1, c_2, c_3 \) does this equation hold?

(a) \( c_1 = 2, c_2 = 2, c_3 = -4 \)
(b) \( c_1 = 1, c_2 = -1, c_3 = 1 \)
(c) \( c_1 = 2, c_2 = -4, c_3 = 2 \)
(d) \( c_1 = -1, c_2 = 1, c_3 = 1 \)

12. Which of the following conditions on \( k \) ensures that the following matrix is not invertible?

\[ M = \begin{bmatrix} k & -1 \\ -1 & k \end{bmatrix} \]

(a) \( k^2 - 1 = 0 \)
(b) \( k^2 + 1 = 0 \)
(c) \( (k - 1)^2 = 0 \)
(d) \( (k + 1)^2 = 0 \)

13. Consider the Simplex Table below:

\[
\begin{bmatrix}
1 & -4 & 0 & 8 & 0 & 45 \\
0 & 3 & 0 & 5 & 1 & 21 \\
0 & 2 & 1 & 3 & 0 & 12
\end{bmatrix}
\]

To perform the simplex algorithm, which matrix entry should be pivoted in this Simplex Table?

(a) None, we are done. This is the final simplex table
(b) Matrix entry \( a_{1,2} \)
(c) Matrix entry \( a_{2,2} \)
(d) Matrix entry \( a_{3,2} \)
14. Matrix $A$ below represents the number of leafblowers and rakes at the downtown and suburban locations of a hardware store. Matrix $B$ below represents the number of wholesale and retail value of leafblowers and rakes. Which of the matrix products below has an economic interpretation?

$$
A = \begin{bmatrix}
30 & 200 \\
50 & 100 \\
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
100 & 300 \\
5 & 20 \\
\end{bmatrix}
$$

(a) $AB$

(b) $AB^T$

(c) $A^TB$

(d) $A^TB^T$

(e) All of the above.

15. Find $x$ and $y$ so that the statement below is true.

$$
\begin{bmatrix}
x & y \\
1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
 2 & -2 \\
 4 & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
 4 & -2 \\
-2 & -2 \\
\end{bmatrix}
$$

(a) $x = 1, y = 1/2$

(b) $x = 2, y = 2$

(c) $x = 1, y = 1$

(d) There are no values of $x$ and $y$ that make the statement true.

(e) The correct answer is not here.
Consider the information below for the following True/False Questions. $\mathbf{u}_1 = \langle 2, -1, 2 \rangle$, $\mathbf{u}_2 = \langle -1, 2 \rangle$, and $\mathbf{u}_3 = \langle 6, -3, 6 \rangle$.

16. The vectors $\mathbf{u}_1$ and $\mathbf{u}_3$ are parallel vectors.
   
   (a) = TRUE  (b) = FALSE

17. The vector $\mathbf{u}_2$ must be drawn in the second quadrant.

   (a) = TRUE  (b) = FALSE

18. The length of vector $\mathbf{u}_1$ is $3\sqrt{2}$

   (a) = TRUE  (b) = FALSE