For questions 1, use the Simplex Tables below:

\[
A = \begin{bmatrix}
1 & -2 & -1 & 3 & 0 & 0 & 0 & 0 \\
0 & 4 & 3 & 4 & 1 & 0 & 0 & 16 \\
0 & -2 & -1 & 5 & 0 & 1 & 0 & 4 \\
0 & 4 & 5 & 2 & 0 & 0 & 1 & 12 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1 & 3 & 1 & -3 & 0 & 27 \\
0 & 0 & 7 & -2 & 1 & 17 \\
0 & 2 & 1 & -4 & 0 & 8 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & -3 & -5 & 0 & 0 & 0 \\
0 & -4 & -1 & 1 & 0 & 7 \\
0 & 1 & 4 & 0 & 1 & -8 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
1 & 5 & 0 & 0 & 2 & 0 & 0 & 25 \\
0 & 6 & 0 & 1 & 0 & 3 & 6 & 18 \\
0 & 2 & 1 & 0 & 6 & 0 & 1 & 24 \\
0 & -2 & 0 & 0 & -1 & 5 & 2 & 7 \\
\end{bmatrix}
\]

\[
E = \begin{bmatrix}
1 & -4 & 0 & -8 & 0 & 45 \\
0 & 3 & 0 & 5 & 1 & 20 \\
0 & 2 & 1 & 3 & 0 & 9 \\
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
1 & 0 & -5 & -1 & 4 & 98 \\
0 & 0 & 2 & 8 & 0 & 1 \\
0 & 1 & 3 & -4 & -6 & 6 \\
\end{bmatrix}
\]

1) (2pt) Which Simplex Table above will not produce a maximum and why?

\[\text{B - THERE IS NO PIVOT POINT UNDER THE -3}\]

2) (2pt) Which table is not produced from a system that meets the criteria for the Simplex Algorithm and how do you know?

\[\text{C - ONE OF THE CONSTRAINTS IS NOT LESS THAN OR EQUAL TO A POSITIVE NUMBER}\]

3) (1pt) Which should be pivoted on the entry 3?  \[E\]

4) (1pt) Which could represent an Initial Simplex Table?  \[A\]

5) (2pt) What is the Basic Feasible Solution for Table \[E\]?

\[0, 9, 0, 20\]  \[\text{MAX 45}\]

6) (2pt) Which could represent a Final Simplex Table and how do you know?

\[\text{D NO NEGATIVE ENTRIES ON THE TOP ROW}\]
7') t) Given \( \mathbf{C} = \begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix} \), find \( [I - \mathbf{C}]^{-1} \) using the \( [A | I] \) method.

\[
[I - \mathbf{C}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -a & -1 \end{bmatrix}
\]

\[
\begin{bmatrix} -1 & 0 \\ -a & -1 \end{bmatrix} \rightarrow [R_1 \rightarrow R_1, \begin{bmatrix} 1 & 0 \\ -a & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}] R_1 + R_2
\]

\[
\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -a & 0 \end{bmatrix}
\]

\[
[I - \mathbf{C}]^{-1} = \begin{bmatrix} -1 & 0 \\ -a & -1 \end{bmatrix}
\]

8) (5pt) Find the determinant of \( \mathbf{D} \) using cofactor expansion. Clearly indicate the row or column used and clearly state the expansion equation produced.

\[
\mathbf{D} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & a & 3 \\ -2 & 1 & 0 \end{bmatrix}
\]

\[
\det(\mathbf{D}) = 2(-1)^{1+1} \det \begin{bmatrix} a & 3 \\ 0 & 1 \end{bmatrix} + 0 + -1(-1)^{1+3} \det \begin{bmatrix} 0 & a \\ -2 & 1 \end{bmatrix}
\]

\[
= 2(-3) + -1(2a)
\]

\[
\det(\mathbf{D}) = -6 - 2a
\]
9) The Island of Bliss has an economy with three interdependent sectors, tourism (to) transportation (tr) and services (sv). Each dollar worth of tourism produced requires an input of $.30 from tourism, $.40 from transportation and $.30 from services. Each dollar of transportation produced requires $.20 of tourism, $.40 of transportation and $.20 of services. Each dollar of services produced requires $.55 of tourism, $.05 of transportation and $.15 of services.

a) (3pt) Write the consumption matrix for this economy.

\[
\begin{bmatrix}
  .3 & .2 & .55 \\
  .4 & .4 & .05 \\
  .3 & .2 & .15 \\
 1.00 & .80 & .70
\end{bmatrix}
\]

b) (2pt) Which, if any, of the sectors are profitable? How do you know?

TRANSPORTATION & SERVICES: THE COLUMNS SUM TO LESS THAN $1.

c) (1pt) Which, if any, of the sectors have surplus production to meet external demand?

TRANSPORTATION & SERVICES

[d) (1pt) Who is the biggest user of the tourism industry?

SERVICES

e) (2pt) What is the first indication that this economy is productive?

\[
[I - C]^{-1} \text{ exists & all entries are positive}
\]

f) (5pt) If there is an external demand for $30 million in tourism, $40 million in transportation and $15 million in services, how much, to the nearest million, will each sector have to produce to meet the demand? Show your equation setup for full credit and state your conclusion.

\[
[I - C]^{-1} \begin{bmatrix}
  30 \\
  40 \\
  15
\end{bmatrix} \times 10^6
\]

Tourism must produce $230 million, transportation $233 million and services $154 million to meet external demand.
The Charlie Chocolate Company makes four types of candy, Dreamy Dark (DD), Chunky Choc (CC), Mellow Milk (MM) and Wild White (WW). They stock their four local stores with these candies. Matrix $A$ below shows the number of pounds of each type of chocolate at each location.

$$
A = \begin{bmatrix}
DD & 50 & 30 & 75 & 40 \\
CC & 50 & 30 & 70 & 40 \\
MM & 65 & 30 & 75 & 40 \\
WW & 10 & 30 & 20 & 20 \\
\end{bmatrix}
$$

Pounds

a) (5pt) The wholesale cost of Dreamy Dark is $7.50/lb, Chunky Choc $6.50/lb, Mellow Milk $7.00/lb and Wild White $8.00/lb. Write a matrix operation that will produce the total wholesale amount at each location and show the resulting matrix. *Clearly label your resulting matrix and name it $B$."

\[
\text{WHOLESALE} = \begin{bmatrix}
7.50 \\
6.50 \\
7.00 \\
8.00 \\
\end{bmatrix}
\]

\[
A^\top \times \text{WHOLESALE} = \begin{bmatrix}
1235 \\
870 \\
1702.50 \\
1000 \\
\end{bmatrix}
\]

\[
\text{WHOLESALE} \quad #
\]

\[
\left(\text{Loc x Type}\right) \times \left(\text{Type x$\#$}\right)
\]

\[
\begin{bmatrix}
\text{ Loc 1} \\
\text{ Loc 2} \\
\text{ Loc 3} \\
\text{ Loc 4} \\
\end{bmatrix}
\]

\[
= B
\]

or

\[
\text{WHOLESALE} = \begin{bmatrix}
7.50, 6.50, 7.00, 8.00 \\
\end{bmatrix}
\]

\[
\text{WHOLESALE} \cdot A = B = \# \begin{bmatrix}
1235, 870, 1702.50, 1000 \\
\end{bmatrix}_{\text{Whk}}.
\]
b) (5pt) The company decided to test the market by pricing the candy differently at each location. At Location 1 they put a profit of $8.00 a pound on all chocolate, at Location 2 a profit of $7.50 a pound, at Location 3 a profit of $9.00 a pound and at Location 4 a profit of $8.50 a pound. Write a matrix operation that will show the total profit for each type of candy if all inventory is sold under this plan and show the resulting matrix. Clearly label your resulting matrix and name it \( C \).

\[
\begin{bmatrix}
\text{Loc 1} & 8 \\
\text{Loc 2} & 7.50 \\
\text{Loc 3} & 9 \\
\text{Loc 4} & 8.50
\end{bmatrix} = P
\]

\[
A \cdot P = \begin{bmatrix}
1640 \\
1595 \\
1760 \\
655
\end{bmatrix} \Rightarrow \begin{bmatrix}
\text{DD} \\
\text{CC} \\
\text{MM} \\
\text{WW}
\end{bmatrix} = C
\]

c) (4pt) Finally the company decided that each location should put a 60% markup on the wholesale price of all candy to produce their profit. Write a matrix operation that shows the profit that would result for each location if they used this plan and show the resulting matrix. Clearly label your resulting matrix.

\[
\begin{bmatrix}
\text{Loc 1} & 7.41 \\
\text{Loc 2} & 5.22 \\
\text{Loc 3} & 10.21.50 \\
\text{Loc 4} & 6.00
\end{bmatrix} = \begin{bmatrix}
\text{Loc 1} \\
\text{Loc 2} \\
\text{Loc 3} \\
\text{Loc 4}
\end{bmatrix} \cdot \text{B}
\]
11) The manager of a local silkscreen t-shirt company has been given the task of maximizing the company's profit by choosing the best schedule for the number of hours to run each silkscreen press per day. Below is the linear program related to this situation.

\[
PROFIT = 70m_1 + 30m_2 + 90m_3 \\
\text{Subject to: } 2m_1 + 4m_2 + 6m_3 \leq 30 \text{ yds} \\
\text{ } 9m_1 + 5m_2 + 6m_3 \leq 60 \text{ gal} \\
\]

\[P - 70m_1 - 30m_2 - 90m_3 = 0\]

a) (4pt) Set up the Initial Simplex Table for this situation and clearly label your first pivot point.

\[
\begin{array}{ccccccc}
1 & -70 & -30 & -90 & 0 & 0 & 0 \\
0 & 4 & 5 & 6 & 1 & 0 & 36 \\
0 & 2 & 4 & 16 & 0 & 1 & 30 \\
0 & 9 & 5 & 6 & 0 & 0 & 60 \\
\end{array}
\]

b) (2pt) Show the Simplex Table that results from your pivot.

\[
\begin{array}{ccccccc}
1 & -40 & 30 & 0 & 0 & 15 & 0 \\
0 & 2 & 1 & 0 & 1 & -1 & 6 \\
0 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{6} & 5 \\
0 & 7 & 1 & 0 & 0 & -1 & 30 \\
\end{array}
\]

c) (2pt) State the basic feasible solution for the table from part b above.

\[
(0, 0, 5, 6, 0, 30) \quad \text{PROFIT} = 450
\]

d) (3pt) The final basic feasible solution is \((3, 0, 4, 9, 0, 0)\), with a maximum of $570. Write a conclusion for the manager of the t-shirt company. Include in your report any slack that may have occurred.

\[
\text{To maximize profit at$570 they should run machine 1 3 hours, machine 3 4 hours and not run machine 2 at all. There will be 9 gals of material not used.}
\]
For questions 12—15 below use the following matrices:

\[
A = \begin{bmatrix} a & b \\ b & c \\ d & -a \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 4 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 2 \\ -2 & -5 \end{bmatrix} \quad D = \begin{bmatrix} -2 \\ 4 \end{bmatrix} \quad E = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -2 & 4 \\ 3 & 0 & 6 \end{bmatrix}
\]

12) (2pt) Find \(2A + B^t\) if possible, if not possible state why.

\[
2 \begin{bmatrix} a & b \\ b & c \\ d & -a \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2a + 2 & 2b + 5 \\ 2b + 3 & 2c + 4 \\ 2d - 1 & -2a + 6 \end{bmatrix}
\]

13) (2pt) Find \(det(C)[C]^{-1}\) if possible, if not possible state why.

\[
\begin{bmatrix} -5 & -2 \\ 2 & 5 \end{bmatrix}
\]

14) (2pt) Find \(B \times E \times D^{-1}\) if possible, if not possible state why.

**No possible because D is not a square matrix so does not have an inverse.**

15) (3pt) Solve \(E^t \begin{bmatrix} x \\ y \\ z \end{bmatrix} = D\) if possible, if not possible state why.

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} \frac{7}{3} \\ -\frac{11}{2} \\ \frac{20}{9} \end{pmatrix}
\]
16) (2pt) What is the purpose of the determinant?

To determine if the matrix has an inverse (is invertible).

17) (3pt) Name three ways you could show that a matrix does not have an inverse.

1) It is not a square matrix

2) \text{rref} is not \text{I}

3) \text{det} = 0

Two rows are multiples

18) (2pt) What is the product of two inverses?

The identity

\[ A^{-1}B^{-1} = (BA)^{-1} \]