Math 125 Fall 2015 Exam 1
Wednesday, September 30th, 2015

Name: ___________________________  UIN: ___________________________

Circle the section you are registered for:

2pm Lecture 3pm Lecture

A. CODE THE LETTERS BELOW IN QUESTIONS 91 - 96

B. All bags, purses, jackets, and other personal items should be at the front of the room.

C. If you have a question, raise your hand and a proctor will come to you. If you have to use the bathroom, do so NOW. You will not be permitted to leave the room and return during the exam.

D. No cell phones, i-Pods, MP3 players. Turn them off now. If you are seen these items in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece.

E. Be sure to print your proper name clearly at the top of this page. Also circle the section for which you are registered.

F. If you finish early, quietly and respectfully get up and hand in your exam.

G. When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

H. To ensure that you receive full credit, show all of your work.

I. Good luck. You have 60 minutes to complete this exam.
Zone 1
1/1. (15 points) A pastry chef bakes three types of desserts each day using a mixture of flour, sugar, and butter. A batch of brownies uses 3 cups of flour, 6 cups of sugar, and 3 cups of butter. A batch of sugar cookies uses 9 cups of flour, 8 cups of sugar, and 5 cups of butter. A batch of crumble cake uses 3 cups of flour, 2 cups of sugar, and 1 cup of butter. The chef has 150 cups of flour, 180 cups of sugar, and 100 cups of butter available today. Tomorrow, they will have only 45 cups of flour, 100 cups of sugar, and 45 cups of butter available. How many batches of each dessert should they produce each day in order to use up all of the ingredients? If the chef bakes $x$ batches of brownies, $y$ batches of sugar cookies, and $z$ batches of crumble cake, set this problem up as a linear system by filling in the blanks below and then solve the problem and interpret the results.

Today Tomorrow

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>$x$</th>
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Solution and Interpretation:

Solution.

1/2. (15 points) A pastry chef bakes three types of desserts each day using a mixture of flour, sugar, and butter. A batch of brownies uses 4 cups of flour, 8 cups of sugar, and 4 cups of butter. A batch of sugar cookies uses 9 cups of flour, 8 cups of sugar, and 5 cups of butter. A batch of crumble cake uses 6 cups of flour, 4 cups of sugar, and 2 cup of butter. The chef has 295 cups of flour, 360 cups of sugar, and 195 cups of butter available today. Tomorrow, they will have only 150 cups of flour, 240 cups of sugar, and 110 cups of butter available. How many batches of each dessert should they produce each day in order to use up all of the ingredients? If the chef bakes $x$ batches of brownies, $y$ batches of sugar cookies, and $z$ batches of crumb cake, set this problem up as a linear system by filling in the blanks below and then solve the problem and interpret the results.

Today Tomorrow

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Solution and Interpretation:

Solution.
Zone 2
2/1. (15 points) A chocolatier makes three types of chocolate using cocoa butter and cream. A batch of dark chocolate uses 3 liters of cocoa butter, and 2 liters of cream. A batch of white chocolate uses 5 liters of cocoa butter, and 5 liters of cream. A batch of milk chocolate uses 9 liters of cocoa butter, and 11 liters of cream. The chocolatier currently has 39 liters of cocoa butter and 46 liters of cream on hand. How much of each type of chocolate should they make in order to use up all of their ingredients? (Assuming they cannot make partial batches, or a negative number of batches.)

A. Set this problem up as a linear system by filling in the blanks below.

\[ x = \]
\[ y = \]
\[ z = \]

\[ \text{cocoa butter: } \_ \_ \_ \_ x + \_ \_ \_ \_ y + \_ \_ \_ \_ z = \_ \_ \_ \_ \]
\[ \text{cream: } \_ \_ \_ \_ x + \_ \_ \_ \_ y + \_ \_ \_ \_ z = \_ \_ \_ \_ \]

B. Find how many batches of each type of chocolate they should make in order to use up all of their ingredients.

Solution.

2/2. (15 points) A chocolatier makes three types of chocolate using cocoa butter and cream. A batch of dark chocolate uses 10 liters of cocoa butter, and 5 liters of cream. A batch of white chocolate uses 4 liters of cocoa butter, and 3 liters of cream. A batch of milk chocolate uses 22 liters of cocoa butter, and 9 liters of cream. The chocolatier currently has 158 liters of cocoa butter and 66 liters of cream on hand. How much of each type of chocolate should they make in order to use up all of their ingredients? (Assuming they cannot make partial batches, or a negative number of batches.)

A. Set this problem up as a linear system by filling in the blanks below.

\[ x = \]
\[ y = \]
\[ z = \]

\[ \text{cocoa butter: } \_ \_ \_ \_ x + \_ \_ \_ \_ y + \_ \_ \_ \_ z = \_ \_ \_ \_ \]
\[ \text{cream: } \_ \_ \_ \_ x + \_ \_ \_ \_ y + \_ \_ \_ \_ z = \_ \_ \_ \_ \]
cocoa butter:  _____ x+  _____ y+  _____ z  =  _____
cream:  _____ x+  _____ y+  _____ z  =  _____

B. Find how many batches of each type of chocolate they should make in order to use up all of their ingredients.

Solution.
Zone 3
3/1. (5 points) Which of the following matrices are in row echelon form?

\[
M = \begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \\
N = \begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}, \\
P = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 1 & 5 \\
0 & 1 & 3 & 2
\end{bmatrix}
\]

A. ★ None of the matrices are in row echelon form

B. Only \( M \)

C. Only \( M \) and \( N \)

D. All of them - \( M, N, \) and \( P \)

Solution.  lalala

3/2. (5 points) Which of the following matrices are in row echelon form?

\[
M = \begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \\
N = \begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}, \\
P = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 1 & 5 \\
0 & 1 & 3 & 2
\end{bmatrix}
\]

A. None of the matrices are in row echelon form

B. ★ Only \( M \)

C. Only \( M \) and \( N \)

D. All of them - \( M, N, \) and \( P \)

Solution.  lalala

3/3. (5 points) Which of the following matrices are in row echelon form?

\[
M = \begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \\
N = \begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \\
P = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 3 & 2 \\
0 & 0 & 1 & 5
\end{bmatrix}
\]

8
A. None of the matrices are in row echelon form

B. Only $M$

C. ★ Only $M$ and $N$

D. All of them - $M$, $N$, and $P$

Solution.  lalala
4/1. (5 points) Which of the following matrices are in reduced row echelon form?

\[ M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

A. Only \( M \)
B. Only \( M \) and \( N \)
C. All of them - \( M, N, \) and \( P \)
D. ★ None of the matrices are in reduced row echelon form

**Solution.** lalala

4/2. (5 points) Which of the following matrices are in reduced row echelon form?

\[ M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

A. Only \( M \)
B. Only \( M \) and \( N \)
C. ★ Only \( M \) and \( P \)
D. All of them - \( M, N, \) and \( P \)
E. None of the matrices are in reduced row echelon form

**Solution.** lalala

4/3. (5 points) Which of the following matrices are in reduced row echelon form?

\[ M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]
A. Only $M$

B. $\star$ Only $M$ and $N$

C. Only $M$ and $P$

D. All of them - $M$, $N$, and $P$

E. None of the matrices are in reduced row echelon form

**Solution.**  lalala
5/1. (5 points) Which of the following statements are true? A homogenous linear system:

I. can not be inconsistent.
II. can have exactly one solution.
III. can not have exactly three solutions.
IV. can have infinitely many solutions.

A. Only II and IV
B. Only I
C. All except III
D. ★ All of the statements are true.

Solution.

5/2. (5 points) Which of the following statements are true? A homogenous linear system:

I. can be inconsistent.
II. can have exactly one solution.
III. can have two different solutions.
IV. can have infinitely many solutions.

A. Only I
B. ★ Only II and IV
C. All except III
D. All of the statements are true.

Solution.

5/3. (5 points) Which of the following statements are true? A homogenous linear system:
I. may have no solution.
II. may have just one solution.
III. may have exactly two solutions.
IV. may have infinitely many solutions.

A. ★ Only II and IV
B. Only I
C. All except III
D. All of the statements are true.

Solution.
6/1. (5 points) Which of the following statements are true?

I. The row rank of \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
6 & 8 & 10 & 12
\end{bmatrix}
\] is 2.

II. The row rank of \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 6 & 8
\end{bmatrix}
\] is 2.

III. The row rank of \[
\begin{bmatrix}
2 & 2 & 2 & 5 \\
-2 & -2 & -2 & 3
\end{bmatrix}
\] is 1.

A. ★ Only I is true.

B. Only I and II are true.

C. All of the above are true.

D. All except II are true.

**Solution.** lala

6/2. (5 points) Which of the following statements are true?

I. The row rank of \[
\begin{bmatrix}
4 & 3 & 2 & 1 \\
8 & 7 & 6 & 5 \\
12 & 10 & 8 & 6
\end{bmatrix}
\] is 2.

II. The row rank of \[
\begin{bmatrix}
3 & 6 & 2 & 4 \\
3 & 6 & 2 & 4 \\
6 & 12 & 4 & 8
\end{bmatrix}
\] is 2.

III. The row rank of \[
\begin{bmatrix}
2 & 2 & 2 & 5 \\
-2 & -2 & -2 & 3
\end{bmatrix}
\] is 2.

A. Only I is true.

B. Only I and II are true.

C. All of the above are true.

D. ★ All except II are true.

**Solution.** lala
7/1. (5 points) Consider the linear system below. What relation between $a$, $b$, and $c$ would ensure that the system is consistent?

\[
\begin{align*}
3x + 3y + 3z &= a \\
x + y + z &= b \\
2x + 2y + 2z &= c
\end{align*}
\]

A. $c - 2a = 0$

B. $3a + b + 2c = 0$

C. ★ $a = 3b, c = 2b$

D. Correct answer is not here.

**Solution.** lala

7/2. (5 points) Consider the linear system below. What relation between $a$, $b$, and $c$ would ensure that the system is consistent?

\[
\begin{align*}
2x + 2y + 2z &= a \\
3x + 3y + 3z &= b \\
x + y + z &= c
\end{align*}
\]

A. $a - 2c = 0$

B. ★ $3c = b, 2c = a$

C. $2a + 3b + c = 0$

D. Correct answer is not here.

**Solution.**

7/3. (5 points) Consider the linear system below. What relation between $a$, $b$, and $c$ would ensure that the system is consistent?
\[ x + y + z = a \]
\[ x + y + z = b \]
\[ 2x + 2y + 2z = c \]

A. \[ c - 2a = 0 \]

B. \[ a + b - c = 0 \]

C. \[ \star a = b, c = 2a \]

D. The correct answer is not here.

Solution.
8/1. (5 points) If a system is inconsistent, then which of the following must be true?

I. The REF of the augmented matrix must have a row of all 0.
II. The REF of the augmented matrix has a contradicting equation.
III. The REF of the coefficient matrix has a row of all 0.

A. Exactly one of the statements I, II, III is true.
B. Both I and II are true.
C. ★ Both II and III are true.
D. All of the statements I, II, III are true.

Solution.

8/2. (5 points) If a system is inconsistent, then which of the following must be true?

I. If the REF of the augmented matrix has a row of all 0, it will be placed at the bottom of the matrix.
II. The REF of the augmented matrix has a contradicting equation.
III. The REF of the coefficient matrix has a row of all 0.

A. Exactly one of the statements I, II, III is true.
B. Both I and II are true.
C. Both II and III are true.
D. ★ All of the statements I, II, III are true.

Solution.
9/1. (5 points) Which of the following row operations are required to move the matrix below into reduced row echelon form?

\[
\begin{bmatrix}
1 & 2 & 6 & 4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 5 \\
\end{bmatrix}
\]

A. \( R'_2 = R_2 - R_1, R'_1 = R_1 - 2R_2 \)

B. \( R'_1 = R_1 - 3R_3, R'_2 = R_2 - 3R_3, R'_3 = R_1 - R_2 \)

C. \( R'_2 = R_2 - 3R_3, R'_1 = R_1 - 6R_3, R'_1 = R_1 - 2R_2 \)

D. The correct answer is not here

**Solution.**
10/1. (5 points) The row rank of a matrix might change between REF and RREF.

A. True
B. ★ False

Solution.

10/2. (5 points) The row rank of a matrix cannot change between REF and RREF.

A. ★ True
B. False

Solution.
11/1. (5 points) Which of these graphs is the feasibility region for the following system of linear inequalities?

\[
\begin{align*}
  x + 2y & \geq 6 \\
  2x + y & \leq 6 \\
  x, y & \geq 0
\end{align*}
\]

I. II. III. IV.

A. Graph I.  
B. ★ Graph II.  
C. Graph III.  
D. Graph IV.  
E. The correct answer is not here.

Solution.


11/2. (5 points) Which of these graphs is the feasibility region for the following system of linear inequalities?
\[ x + 2y \geq 6 \\
2x + y \geq 6 \\
x, y \geq 0 \]

III.

A. Graph I.
B. Graph II.
C. Graph III.
D. ★ Graph IV.
E. The correct answer is not here.

Solution.
12/1. (5 points) You are selling atomic, awesome and average cupcakes at your store "Cupcakes! TM". Each atomic cupcake uses 4 oz of sugar and 2 oz of secret ingredient, and sells for $6. Each awesome cupcake uses 2 oz of sugar and 1 oz of secret ingredient, and sells for $2. Each average cupcake uses 1 oz of sugar and 1 oz of secret ingredient and sells for $1. You have 100 oz of sugar and 60 oz of secret ingredient in stock. To maximize revenue you run the simplex algorithm on the following linear program:

\[
\begin{align*}
  x &= \# \text{ of atomic cupcakes} \\
  y &= \# \text{ of awesome cupcakes} \\
  z &= \# \text{ of average cupcakes} \\

  R &= 6x + 2y + z \quad \implies \quad R - 6x - 2y - z = 0 \\
  4x + 2y + z &\leq 100 \quad \implies \quad 3x + 2y + z + s_1 = 100 \\
  2x + y + z &\leq 60 \quad \implies \quad 2x + y + z + s_2 = 60
\end{align*}
\]

When trying to maximize revenue over this set of constraints, are any ingredients left over?

A. ★ There are 10 oz of secret ingredient left over.

B. There are 25 oz of secret ingredient left over.

C. There are 10 oz of sugar left over.

D. There are 25 oz of sugar left over.

E. The correct answer is not here.

Solution.
13/1. (5 points) What is the geometric shape of this solution set? Choose the BEST answer.

\[ \{(t + s, 2s, 4) | t \in \mathbb{R}\} \]

A. The empty set.

B. A single point.

C. A line.

D. ★ A plane.

E. It has infinitely many solutions.

Solution.