Math 347

Midterm 1  Midterm 2  Midterm 3  Post-Midterm 3

- Logic and proofs
  - Sets and functions
  - More functions
  - The real number axioms
  - Divisibility and the Euclidean algorithm
  - Induction and well-ordering
  - Cardinality of sets

Logic and proofs

- Propositional logic
  - Truth tables, \( \land, \lor, \neg, \rightarrow, \leftrightarrow \)
  - Negation rules, converse, contrapositive
- Types of proofs and basic proofs
  - Direct proofs and proof by contrapositive (e.g., divisibility properties and inequalities of real numbers)
  - Proof by contradiction (e.g., in finite sets, \( \exists \) is vacuous)
- Quantifiers
  - \( \forall, \exists \) and rules for \( \neg \)

Divisibility and the Euclidean algorithm

- The division algorithm and congruences
  - Statement of the division algorithm
  - Definition of congruence and equivalent properties
  - Arithmetic modulo \( n \)
• GCDs and the Euclidean algorithm
  - gcds
  - the Euclidean algorithm
  - Bezout’s identity and linear Diophantine equations
  - Applications of Bezout’s identity to prime numbers
  - RSA cryptosystem

Basic sets and functions

• Sets
  - terminology, notation, subsets
  - union, intersection, complement
  - set theoretic algebra

• Functions
  - basic definitions
  - injectivity, surjectivity, bijectivity
  - Finite sets and the pigeonhole principle
  - composition of functions

Induction

• Basic induction
  - method and examples

• Strong induction
  - method and examples

• Well-orderedness
  - N is well-ordered
  - well-ordered \(\Rightarrow\) induction
  \(\Rightarrow\) division algorithm

More sets and functions

• More sets
  - Cartesian products
- Power sets
- Unions and intersections of indexed collections of sets
- The Cantor set

• More on functions
  - inverse functions and relation to bijections
  - inverse images of sets under (arbitrary) functions

Equivalence relations

• Relations and equivalence relations
  - basic definitions and examples
  - equivalence classes

• Partitions
  - relation to equivalence relations

• Well-definition
  - modular arithmetic again
  - functions on equivalence classes

Cardinality

• Basic definitions
  - $|A| \leq |B|$, $|A| = |B|$, $|A| < |B|

• Denumerable sets
  - examples: $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{N} \times \mathbb{N}$, certain unions
  - the smallest infinity

• Bigger sets
  - $[0,1]$ is uncountable
  - Cantor–Schröder–Bernstein and applications
    (e.g. $|UR| = |\text{Cantor set}| = |[0,1]|$)
- Cardinalities of power sets

**The Real Numbers and Sequences of Real Numbers**

- **The real numbers**
  - Construction of \( \mathbb{R} \)
  - \( \inf \), \( \sup \), and the least upper bound property

- **Sequences of real numbers**
  - Limits and convergence of sequences
  - Bounded sequences, the monotone convergence theorem, and the squeeze theorem
  - Limit laws
  - Subsequences and the Bolzano–Weierstrass theorem
  - Cauchy sequences and the completeness of \( \mathbb{R} \)

- **Infinite series**
  - Definition and examples
  - The comparison test and the ratio test