Topics: The Cantor-Schröder-Bernstein Theorem.

**Theorem (Cantor-Schröder-Bernstein)** If $A$ and $B$ are sets such that $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.

*Note: This may look obvious, but it's not! (It is obvious if $A$ and $B$ are finite.) Unravelling its definitions, the theorem says that if $f : A \rightarrow B$ and $g : B \rightarrow A$ are injections, then $g \circ f : A \rightarrow B$ is a bijection.*

**Proof.**

*Example* Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Then $A$ is countable, since $A \subseteq \mathbb{N} \Rightarrow |A| \leq |\mathbb{N}|$.

Now consider the map $f : \mathbb{N} \rightarrow A$ given by $f(n) = 2n$ for any $n \in \mathbb{N}$, then $2n = 2m \Rightarrow n = m$, so $f$ is injective. Thus $|A| \leq |\mathbb{N}|$.

By Cantor-Schröder-Bernstein, we conclude $|A| = |\mathbb{N}|$.

Hence, find a bijection $f : \mathbb{N} \rightarrow A$.

Workshop time!