Let $U$ be a set and let $A$ and $B$ be subsets of $U$.

- The complement of $A$ in $U$, written $A^c$ or $U \setminus A$, is
  \[ A^c = U \setminus A = \{ x \in U : x \notin A \} \]
- The complement of $A$ relative to $B$, written $B \setminus A$, is
  \[ B \setminus A = \{ x \in B : x \notin A \} \]
- The union of $A$ and $B$, written $A \cup B$, is
  \[ A \cup B = \{ x \in U : x \in A \text{ or } x \in B \} \]
- The intersection of $A$ and $B$, written $A \cap B$, is
  \[ A \cap B = \{ x \in U : x \in A \text{ and } x \in B \} \]

In picture:

![Venn Diagram](https://via.placeholder.com/150)
\[ \begin{align*}
U &= \mathbb{Z}, \quad A = \{2\mathbb{Z}\}, \quad B = \{0, 1, 2, 3, 4\} \\
U \setminus A &= \{ x \in \mathbb{Z} : x \text{ is odd} \} \\
B \setminus A &= \{ 1, 3 \} \\
A \cap B &= \{ x \in \mathbb{Z} : x < 0 \text{ or } x > 4 \} \\
A \cup B &= \{ x \in \mathbb{Z} : x \text{ is even or } x = 1 \text{ or } x = 3 \} \\
A \cap B &= \{ 0, 2, 4 \}
\end{align*} \]

Then let \( A, B, C \) be sets.

1. \( \emptyset \cup A = A \) and \( \emptyset \cap A = \emptyset \)
2. \( A \cap B \subseteq A \subseteq A \cup B \)
3. \( A \cup B = B \cup A \) and \( A \cap B = B \cap A \)
4. \( (A \cup B) \cap C = A \cup (B \cap C) \) and \( (A \cap B) \cup C = A \cap (B \cup C) \)
5. \( A \cup A = A = A \cap A \)
6. \( A \subseteq B \Rightarrow A \cup C \subseteq B \cup C \) and \( A \cap C \subseteq A \cap C \)

**Proof**

1. Assume \( A \subseteq B \). We first show \( A \cup C \subseteq B \cup C \).

   Take \( x \in A \cup C \), we want to show \( x \in B \cup C \).

   Since \( x \in A \cup C \), either \( x \in A \) or \( x \in C \). If \( x \in A \), then \( x \in B \) since \( A \subseteq B \), hence \( x \in B \cup C \).

   If \( x \in C \), then we also have \( x \in B \cup C \). So \( x \in B \cup C \) in either case.

   Now we show that \( A \cap C \subseteq B \cap C \).

   Take \( x \in A \cap C \). Then \( x \in A \) and \( x \in C \). Since \( A \subseteq B \), we see that \( x \in B \) and \( x \in C \), so \( x \in B \cap C \).

2. Let \( A, B \) be subsets of \( \mathbb{Z} \subseteq U \).

   1. \( U \setminus (A \cap B) = (U \setminus A) \cup (U \setminus B) \)
   2. \( U \setminus (A \cup B) = (U \setminus A) \cap (U \setminus B) \)
   3. \( U \setminus (U \setminus A) = A \)
   4. \( A \setminus B = A \cap B \)
   5. \( A \subseteq B \Leftrightarrow (U \setminus B) \subseteq (U \setminus A) \)
Rule 2: Venn diagrams can help you remember/picture these kinds of statements.

Proof of 2: For \( x \in U \), we have

\[
\begin{align*}
    x \in U \setminus (A \cup B) & \iff x \notin A \cup B \\
    & \iff x \text{ is not an element of } A \text{ or } B \\
    & \iff x \text{ is not an element of } A \text{ and } x \text{ is not an element of } B \\
    & \iff x \in (U \setminus A) \cap (U \setminus B).
\end{align*}
\]

Rule 1 and 2 are called de Morgan's Laws.

Campus with de Morgan's Laws from the beginning.

Thus (Distributive Law for sets) Let \( A, B, C \) be sets.

1. \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)
2. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

Proof of 2: First take \( x \in A \cup (B \cap C) \).

Then \( x \in A \) or \( x \in B \cap C \). If \( x \in A \), then \( x \in A \cup B \) and \( x \in A \cup C \), so \( x \in (A \cup B) \cap (A \cup C) \).

If \( x \in B \cap C \), then \( x \in B \) and \( x \in C \), so \( x \in A \cup B \) and \( x \in A \cup C \), hence \( x \in (A \cup B) \cap (A \cup C) \).

This shows \( A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \).

Now take \( x \in (A \cup B) \cap (A \cup C) \).
Then \( x \in A \cup B \) and \( x \in A \cup C \). If \( x \in A \) then we must have \( x \in B \) or \( x \in C \), so \( x \in A \cup (B \cap C) \) again. This shows \((A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)\).

Since \( A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \) and \((A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)\), we have \[ A \cup (B \cap C) = (A \cup B) \cap (A \cup C). \]

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**Functions**

Definition Let \( A \) and \( B \) be sets. A function from \( A \) to \( B \) is a rule that assigns one (and only one) element of \( B \) to each element of \( A \). If \( f \) is a function from \( A \) to \( B \), we write \( f : A \rightarrow B \) and \( f(a) \in B \) for the element assigned to \( a \in A \).

The domain of \( f \), written \( \text{dom}(f) \), is \( A \), and the codomain is \( B \). The image or range of \( f \), written \( \text{im}(f) \) or \( \text{range}(f) \), consisting of all the elements assigned by the rule \( f \).

Diagram:

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\[ A \] \quad \xrightarrow{f} \quad \[ B \]
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\( f(x) \) \quad \( f(y) = f(z) \)