1. Let $A$, $B$, and $C$ be subsets of a set $U$.

   (a) Prove that $U \setminus (A \cup B) = (U \setminus A) \cap (U \setminus B)$.

   (b) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
2. (a) Let $f: A \to B$ be a function. Define what it means for $f$ to be injective and what it means for $f$ to be surjective.

(b) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ that is surjective but not injective. Justify your answer.
3. Use induction to prove that for every integer $n \geq 2$, there are nonnegative integers $x$ and $y$ such that $n = 2x + 3y$. 
4. Prove or disprove the following statement: If $A \subseteq \mathbb{R}$ is well-ordered, then any $B \subseteq A$ is well-ordered.

5. Let $A$ and $B$ be sets. Prove that $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. 
6. For each \( n \in \mathbb{N} \), let

\[ A_n = \left\{ \frac{a}{n} \in \mathbb{Q} : a \in \mathbb{Z} \right\}. \]

(a) Find \( \bigcup_{n \in \mathbb{N}} A_n \). Justify your answer.

(b) Find \( \bigcap_{n \in \mathbb{N}} A_n \). Justify your answer.

(c) Is the collection \( \{ A_n : n \in \mathbb{N} \} \) pairwise disjoint? Justify your answer.