1. State the converse and the contrapositive of each of the following propositions.

(a) If $c \mid a$ and $c \mid b$, then $c \mid (ax + by)$ for all $x, y \in \mathbb{Z}$.

   Converse:

   Contrapositive:

(b) If $f : A \to B$ and $g : B \to C$ are both injective, then $g \circ f : A \to C$ is injective.

   Converse:

   Contrapositive:

2. State the negation of each of the following propositions.

(a) Every element of the set $A$ is either an element of the set $B$ or an element of the set $C$.

(b) For every $\epsilon > 0$, there is $N \in \mathbb{N}$ such that for all $n, m \geq N$, we have $|x_n - x_m| < \epsilon$. 
3. Find all $x, y \in \mathbb{Z}$ such that $57x + 96y = 6$ or prove that none exist.
4. Let $f : A \to B$ and $g : B \to C$ be functions. For each of the following, determine whether or not the statement is true or false. Justify your answers.

(a) If $g \circ f$ is injective, then $f$ is injective.

(b) If $g \circ f$ is injective, then $g$ is injective.

(c) If $g \circ f$ is surjective, then $f$ is surjective.

(d) If $g \circ f$ is surjective, then $g$ is surjective.
5. For each $r \in \mathbb{R}$, let $A_r = \{(x, y) \in \mathbb{R}^2 : xy = r\}$.

(a) Find $\bigcup_{r \in \mathbb{R}} A_r$. Justify your answer.

(b) Is the indexed collection $\{A_r : r \in \mathbb{R}\}$ pairwise disjoint? Justify your answer.

(c) Is the indexed collection $\{A_r : r \in \mathbb{R}\}$ a partition of $\mathbb{R}^2$?
6. For all $n \in \mathbb{N}$, $4 \mid (3^{2n} - 5^n)$.

(a) Prove this using induction.

(b) Prove this using modular arithmetic.
7. Determine whether or not the following relations $R$ satisfy each of the properties of reflexivity, symmetry, and transitivity. Justify your answers.

(a) The relation $R$ on $\mathbb{Z}$ given by $aRb \iff ab$ is even.

(b) The relation $R$ on $\mathcal{P}(\mathbb{N})$ given by $ARB \iff A \cap B = A$. 
8. Let $A$ and $B$ be infinite sets. Prove that $A \cup B$ is denumerable if and only if $A$ and $B$ are both denumerable.
9. (a) Define what it means for a sequence of real numbers \((x_n)_{n=1}^{\infty}\) to have a limit \(L \in \mathbb{R}\).

(b) Use the definition to prove that \(\lim_{n \to \infty} \left(2 + \frac{7}{(n+1)^2}\right) = 2\).
10. Using the definition, prove that if \((x_n)_{n=1}^\infty\) and \((y_n)_{n=1}^\infty\) are Cauchy sequences of real numbers, then \((x_n + y_n)_{n=1}^\infty\) is a Cauchy sequence.
11. Let \((x_k)_{k=1}^{\infty}\) be a sequence of real numbers.

   (a) Define what it means for the infinite series \(\sum_{k=1}^{\infty} x_k\) to converge.

   (b) Prove that if \(\sum_{k=1}^{\infty} x_k\) converges, then \(\lim_{k \to \infty} x_k = 0\).