If $f$ is such that $\int_0^3 f(x) \, dx = -1$, then if $f$ is even, $f(-x) = f(x)$, so
$$\int_{-3}^3 f(x) \, dx = \int_0^3 f(x) \, dx + \int_{-3}^0 f(x) \, dx.$$ We’re given that the latter integral is equal to $-1$, and to evaluate the former integral we let $u = -x$. Then $du = -dx$, and
$$\int_{-3}^3 f(x) \, dx = \int_{-3}^3 f(u) \, du = \int_3^0 f(u) \, du = -1.$$ Then the sum is $-2$ as claimed. Another way to see it is that an even function is symmetric about the $y$-axis, so the signed area under $f$ on $[0, 3]$ is equal to the signed area under $f$ on $[-3, 0]$.

If $f$ is an odd function, then $\int_{-3}^3 f(x) \, dx = \int_0^0 f(x) \, dx + \int_{-3}^3 f(x) \, dx$, but the former integral (using the same method as above) turns out to be equal to $1$, and thus the sum is $0$. Again, symmetry is another way to see that this is the case.

We know that on $[0, \pi]$, $\sin(x) \geq 0$ and $x \geq 0$, and that the minimum value of the two functions is $0$. Thus, the signed area under the graph of $x \sin(x)$ on $[0, \pi]$ will be at least $0$. Also, $x \sin(x)$ achieves its maximum value at $\frac{\pi}{2}$, which means that the integral will be bounded by $\frac{\pi}{2} \pi = \frac{\pi^2}{2}$.

Let $f(x) = \sqrt{4 + x}$. Then the right-hand Riemann sum approximation for $f(x)$ on the interval $[0, 5]$ is equal to $\frac{5}{n} \sum_{k=1}^n \sqrt{4 + 5k/n}$, and taking the limit as $n \to \infty$, we obtain the value $\int_0^5 \sqrt{4 + x} \, dx$. Using substitution we can evaluate this integral, and it equals $\frac{336}{3}$. Notice that we also could have used the function $f(x) = \sqrt{x}$ and the interval $[4, 9]$, as it would have given us the exact same Riemann sum approximation.

\[ \int_4^1 2h(z) - 5 \, dz = -\int_4^1 2h(z) - 5 \, dz = \int_1^4 5 - 2h(z) \, dz = \int_1^4 5 \, dz - 2 \int_1^1 h(z) \, dz = \int_1^4 5z \, dz = 15 - 34 = -19. \]