Solutions to homework due on Friday, 3/30

(4.6: 4a) We want to use Newton’s method to find an approximate value of $10^{1/3}$. That is, we want to find a root of $x^3 - 100$. Beginning with $x_0 = 4.5$, $x_1 = 4.5 - \frac{(4.5)^3 - 100}{3(4.5)^2} = 4.64609$. Continuing, $x_2 = 4.65169$ and $x_3 = 4.65159$. The last two agree to three decimal places, so we’re done.

(4.6: 9) (a) When $x_0 = 2$, $x_1 = 1.2500000$, $x_2 = 1.0250000$, $x_3 = 1.000304878$, and $x_4 = 1.000000046$. They are accurate to 0, 1, 3, and 7 decimal places respectively.

(b) Newton’s method finds $x = 1$ if $x_0 > 0$; it finds $x = -1$ if $x_0 < 0$. It fails if $x_0 = 0$.

(4.6: 16) Look at the graph and use Newton’s method. Use the fact that $h(-x) = h(x)$ to your advantage.

(4.9: 8) The parabola $x^2 - 1$ will have two roots and only one stationary point.

(4.9: 9) Not possible: Rolle’s theorem implies that any such function will have to have a stationary point on the intervals $(a, b)$ and $(b, c)$ where $a, b, c$ are the roots of $f$.

(4.9: 10) $f(x) = x^3 - x + 2$ has one root and two stationary points.

(4.9: 11) $f(x) = \sin(x)$ has infinitely many roots and stationary points.

(4.9: 12) This is really an IVP: If $f'(x) = -1$, then $f(x) = -x + C$, and since $f(0) = 137$, we get that $f(x) = -x + 137$.

(4.9: 15) $f$ does not satisfy the MVT hypothesis on $[-1, 2]$ since it is not differentiable at $x = 0$.

(4.9: 16) Since $f'(x) = 1$ or $f''(x) = -1$, and $(f(2) - f(-1))/(2 - (-1)) = 1/3$, there does not exist a number $c$ in $(-1, 3)$ such that $f''(c) = 1/3$.

(4.9: 17) Yes, since $f$ is differentiable (and therefore continuous) on $[1, 2]$, it satisfies the MVT hypothesis for that interval.

(4.9: 18) By the above, $f$ has to satisfy the conclusion of the MVT since it satisfies the hypothesis for the MVT. On $[1, 2]$, $((f(2) - f(1))/(2 - 1) = 1$, and any $c$ in $(1, 2)$ will do.

(4.7: 1) Use theorem 6 to note that $f(0) = 1$, $f'(0) = 1$, $f''(0) = 2$, $f'''(0) = 6$, $f^{(4)}(0) = 120$, $f^{(5)}(0) = 96$, $f^{(6)}(0) = 0$, and $f^{(131)}(0) = 0$.

(4.7: 2) Use theorem 6 and realize that the only non-zero derivative is $f^{(5)}(1) = 120$. 

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(4.7: 8) If \( f(x) = \sqrt{x} \), then \( f'(x) = 1/(2\sqrt{x}) \) and \( f''(x) = -1/(4x^{3/2}) \). At \( x_0 = 100 \), \( f(100) = 10 \), and \( f'(100) = 1/20 \), and \( f''(100) = -1/4000 \). Thus \( T_1(x) = 10 + 1/20(x - 100) \) and \( T_2(x) = 10 + 1/20(x - 100) - 1/8000(x - 100) \). \( T_1(121) = 11.05 \) and \( T_2(121) = 10.9949 \). Since \( \sqrt{121} = 11 \), \( T_1 \) overestimates by 0.05 and \( T_2 \) underestimates by 0.0051.

(4.7: 17) Linear: \( T_1(x) = 1 + x \). Quadratic: \( T_2(x) = 1 + x + x^2/2 \).

(4.7: 18) Linear: \( T_1(x) = x \) and \( T_2(x) = x \).