Math 461, Fall 2018. Third Midterm Practice

(1) Let $X$ be uniformly distributed on $(0, \pi)$ and $Y = \cos(X - \frac{\pi}{2})$.
   a) Find the probability density function of $Y$.
   b) Find $E(Y)$ and $\text{Var}(Y)$.

(2) Two people agree to meet at a certain location around 12:30 pm.
   One of the people will arrive at a time uniformly distributed between 12:15 and 12:45, and the other person will arrive at a time uniformly distributed between 12:00 and 1:00. Find the probability that the first person to arrive has to wait no longer than 10 minutes.

(3) The joint density of $X$ and $Y$ is given by
   $$f(x, y) = \begin{cases} \frac{1}{3}(x + 1) & \text{if } 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$
   a) Find the conditional density of $Y$ given that $X = \frac{1}{2}$.
   b) Are $X$ and $Y$ independent?
   c) Find $E(X)$ and $\text{Var}(X)$.

(4) Let $X$ and $Y$ be independent exponential random variables with the same parameter $\lambda > 0$.
   a) Find the probability density function of $Z = \frac{2X}{Y}$
   b) Calculate $P(2X \leq 3Y)$.

(5) Suppose that $X$ and $Y$ are independent random variables, $X$ is Poisson with parameter $\lambda$ and $Y$ is Poisson with parameter $\mu$. Find the conditional distribution of the random variable $X$ given that $X + Y = 50$.

(6) Let $N$ be a geometric random variable with parameter $p$. Suppose that the conditional distribution of $X$ given that $N = n$ is the gamma distribution with parameters $n$ and $\lambda$. Find the conditional probability mass function of $N$ given that $X = x$. 