Mathematical Truths:
Experiment, Proof, and Understanding

Nathan M. Dunfield

University of Illinois
What is topology? The study of objects up to rubbery stretching.

What is a topologist?
Someone who can’t tell a coffee cup from a doughnut.
1-manifold: Locally like a line.

2-manifold (surface): Locally like a plane.

3-manifold: Locally like 3-dimensional space.
3-manifolds with boundary a torus:
A 3-manifold is *fibered* if it is swept out by surfaces. For instance, the doughnut is a circle’s worth of disks.
Not every 3-manifold fibers. For instance, this one does not:

**Main Question:** How common is it for a 3-manifold to fiber?
A special kind of 3-manifold:

Revised question: How common is it for such a 3-manifold to fiber?
Coordinates for curves on a surface:

Weights: 1 2 2
Twists: 0 1 -1

Experimental Results:

% of manifolds which fiber

Size of coordinates
Q1: *Does a typical 3-manifold fiber?*

A: No, at least for the type of 3-manifold we've looked at. In particular, the more complicated the manifold, the closer the odds of it fibering is to 0%.
Q1: *Does a typical 3-manifold fiber?*

A: No, at least for the type of 3-manifold we’ve looked at. In particular, the more complicated the manifold, the closer the odds of it fiber ing is to 0%.

Q2: *Why?*
Q1: Does a typical 3-manifold fiber?

A: No, at least for the type of 3-manifold we’ve looked at. In particular, the more complicated the manifold, the closer the odds of it fibering is to 0%.

Q2: Why?

Q3: How can we prove this?
Alternate sampling method

% of manifolds which fiber

Size of coordinates
Two random walks in the plane
Good books about topology:


Original Sources:

- This presentation: http://dunfield.info/preprints/