More on Stokes’ Theorem

1. Let \( \mathbf{F} = \langle y^2, x^2, z^2 \rangle \). Show that

\[
\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}
\]

for any two closed curves as shown lying on a cylinder about the z-axis.

2. Consider the surface \( T \) which is the intersection of the plane \( x+2y+3z = 1 \) with the first octant.

(a) Draw a picture of \( T \).

(b) Use Stokes’ Theorem to evaluate \( \int_{\partial T} \mathbf{F} \cdot d\mathbf{r} \) for \( \mathbf{F} = \langle y, -2z, 4x \rangle \). Here, you should orient \( \partial T \) counterclockwise when viewed from (2, 2, 2).

3. Carefully explain how Green’s Theorem is actually a special case of Stokes’ Theorem.

4. Work the following problem.

20. The magnetic field \( \mathbf{B} \) due to a small current loop (which we place at the origin) is called a magnetic dipole (Figure 18). Let \( \rho = (x^2 + y^2 + z^2)^{1/2} \). For \( \rho \) large, \( \mathbf{B} = \text{curl} (\mathbf{A}) \), where

\[
\mathbf{A} = \begin{pmatrix}
-\frac{y}{\rho^3}, & \frac{x}{\rho^3}, & 0
\end{pmatrix}
\]

(a) Let \( \mathcal{C} \) be a horizontal circle of radius \( R \) with center \((0, 0, c)\), where \( c \) is large. Show that \( \mathbf{A} \) is tangent to \( \mathcal{C} \).

(b) Use Stokes’ Theorem to calculate the flux of \( \mathbf{B} \) through \( \mathcal{C} \).