1. Consider the ellipsoid with implicit equation
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \]

(a) Parameterize this ellipsoid.

(b) Set up, but do not evaluate, a double integral that computes its surface area.

2. Let
\[ \mathbf{r}(u, v) = \langle (2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u \rangle, \]
where \(0 \leq u \leq 2\pi\) and \(0 \leq v \leq 2\pi\).

(a) Sketch the surface parameterized by this function.

(b) Compute its surface area.

3. Consider the surface integral
\[ \iint_{\Sigma} z \, dS \]
where \(\Sigma\) is the surface with sides \(S_1\) given by the cylinder \(x^2 + y^2 = 1\), \(S_2\) given by the unit disk in the \(xy\)-plane, and \(S_3\) given by the plane \(z = x + 1\). Evaluate this integral as follows:

(a) Parameterize \(S_1\) using \((\theta, z)\) coordinates.

(b) Evaluate the integral over the surface \(S_2\) without parameterizing.

(c) Parameterize \(S_3\) in (Des)cartesian coordinates and evaluate the resulting integral using polar coordinates.

4. Let \(C\) be the circle in the plane with equation \(x^2 + y^2 - 2x = 0\).

(a) Parameterize \(C\) as follows. For each choice of a slope \(t\), consider the line \(L_t\) whose equation is \(y = tx\). Then the intersection \(L_t \cap C\) of \(L_t\) and \(C\) contains two points, one of which is \((0, 0)\). Find the other point of intersection, and call its \(x\)- and \(y\)-coordinates \(x(t)\) and \(y(t)\). Compute a formula for \(\mathbf{r}(t) = \langle x(t), y(t) \rangle\). Check your answer with your TA.

(b) Suppose that \(t = \frac{p}{q}\) is a rational number. Show that \(x(p/q)\) and \(y(p/q)\) are also rational numbers. Explain how, by clearing denominators in \(x(p/q) - 1\) and \(y(p/q)\), you can find a triple of integers \(U, V, W\) for which \(U^2 + V^2 = W^2\).

(c) Compute \(\int_C \frac{1}{2} \langle -y, x \rangle \cdot d\mathbf{r}\) using your parameterization above.