Thursday, August 29  

Parametric curves defined using vector arithmetic.

1. Let \( f(x) = x^2 + x - 2 \).

   (a) Graph the equation \( y = f(x) \). (By hand, then check with a calculator if you want.)

   (b) Find the slope and equation of the tangent line to \( y = f(x) \) when \( x = 2 \). Draw the tangent line on your picture.

   (c) Draw a vector in \( \mathbb{R}^2 \) that describes the direction of the line. Find a numeric representation of your vector.

2. Consider the curve given parametrically by

   \[
   \begin{aligned}
   x(t) &= t \\
   y(t) &= t^2 + t - 2 
   \end{aligned}
   \quad \text{for } 0 \leq t < 4.
   \]

   (a) Sketch the curve. How does this graph differ from your graph in Problem 1(a)?

   (b) Consider the vectors formed by the pair \( (x(t), y(t)) \). Anchoring the vectors at the origin, sketch on your graph the vectors at time \( t = 0, 1, 2, 3 \).

   (c) Now consider the vectors formed by \( (x'(t), y'(t)) \). Evaluate \( x'(t), y'(t) \) at time \( t = 2 \), what does the vector represent? Hint: Graph it on the curve at the point \( (x(2), y(2)) \).

   (d) Imagine that the curve is the path of a moving particle. What is the speed of the particle when \( t = 2 \)?

3. (a) Sketch the vector emanating from the origin ending at the point \( (-5, 2) \) in \( \mathbb{R}^2 \).

   (b) On the same graph and using the “head-to-tail” geometric addition method, draw the vector \( (-5, 2) + (3, -1) \).

   (c) Do the same for \( (-5, 2) + 2(3, -1) \).

   (d) Do the same for \( (-5, 2) + (-1)(3, -1) \).

   (e) If we allow the scalar multiplying the vector \( (3, -1) \) to vary, what geometric object is described by the parametric equation \( (-5, 2) + t(3, -1) \) for all \( t \)?

4. Consider the set of points in \( \mathbb{R}^3 \) defined by the parametric equation

   \[ I(t) = (-5 + 2t, 2 + 3t, 1 - t) \quad \text{for all } t. \]

   (a) Using the properties of vector arithmetic, factor \( I(t) \) into the form \( p + tv \) where \( p \) and \( v \) are vectors in \( \mathbb{R}^3 \).

   (b) Using the factored form (and your technique from Problem 3) sketch this object in \( \mathbb{R}^3 \). Geometrically, what does this parametric equation describe?

   (c) Why is the vector \( v \) in your factored form referred to as the direction vector?

5. Let \( a = (-\sqrt{3}, 0, -1, 0) \) and \( b = (1, 1, 0, 1) \) be vectors in \( \mathbb{R}^4 \).

   (a) Find the distance between the points \( (-\sqrt{3}, 0, -1, 0) \) and \( (1, 1, 0, 1) \).

   (b) Find the angle between \( a \) and \( b \).