1. Let $S$ be the portion of the plane $x + y + z = 1$ which lies in the positive octant.

   (a) Draw a picture of $S$.

   **Solution.** The picture is shown below.

   ![Picture of S](image1)

   (b) Find a parametrization $\mathbf{r}: D \to S$, being sure to clearly indicate the domain $D$. Check your answer with the instructor.

   **Solution.** One can use the parametrization $\mathbf{r}(u, v) = (u, v, 1 - u - v)$ with the domain $D$ given by $D = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1 - u\}$.

2. Consider the surface $S$ which is the part of $z + x^2 + y^2 = 1$ where $z \geq 0$.

   (a) Draw a picture of $S$.

   **Solution.** The picture is shown below.

   ![Picture of S](image2)
(b) Find a parametrization \( r : D \rightarrow S \). Check your answer with the instructor.

**Solution.** One can use the parametrization \( r(r, \theta) = (r \cos \theta, r \sin \theta, 1 - r^2) \) with the domain \( 0 \leq r \leq 1, \ 0 \leq \theta \leq 2\pi \).

3. Let \( S \) be the surface given by the following parametrization. Let \( D = [-1, 1] \times [0, 2\pi] \) and define \( r(u, v) = (u \cos v, u \sin v, v) \).

(a) Consider the vertical line segment \( L = \{u = 0\} \) in \( D \). Describe geometrically the image of \( L \) under \( r \).

**Solution.** The image of \( u = 0 \) under \( r \) is a line segment \((0, 0, v)\) where \( 0 \leq v \leq 2\pi \).

(b) Repeat for the vertical segments where \( u = -1 \) and \( u = 1 \).

**Solution.** When \( u = 1 \), the image \( r(1, v) = (\cos v, \sin v, v) \) is a helix with \( 0 \leq v \leq 2\pi \), and so is \( u = -1 \). Thus the images of \( u = 1 \) and \( u = -1 \) form the double helix.

(c) Use your answers in (a) and (b) to make a sketch of \( S \).

**Solution.** The picture is shown below.

4. Consider the ellipsoid \( E \) given by \( \frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1 \).

(a) Draw a picture of \( E \).

**Solution.** The picture is shown below.
(b) Find a parametrization of $E$. Hint: Find a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ which takes the unit sphere $S$ to $E$, and combine that with our existing parametrization of the plain sphere $S$.

**Solution.** One can use the following parametrization

$$\mathbf{r}(\theta, \phi) = (3 \sin \theta \cos \phi, 2 \sin \theta \sin \phi, \cos \theta)$$

with the domain $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. 