Surface Parameterpalooza

1. Let \( S \) be the portion of the plane \( x + y + z = 1 \) which lies in the positive octant.
   
   (a) Draw a picture of \( S \).
   
   (b) Find a parameterization \( \mathbf{r}: D \to S \), being sure to clearly indicate the domain \( D \). Check your answer with the instructor.
   
   (c) Use your answer in (b) to compute the area of \( S \) via an integral over \( D \).
   
   (d) Check your answer in (c) using only things you learned in the first few weeks of this class.

2. Consider the surface \( S \) which is the part of \( z + x^2 + y^2 = 1 \) where \( z \geq 0 \).
   
   (a) Draw a picture of \( S \).
   
   (b) Find a parameterization \( \mathbf{r}: D \to S \). Check your answer with the instructor.

3. Let \( S \) be the surface given by the following parameterization. Let \( D = [-1, 1] \times [0, 2\pi] \) and define
   
   \[ \mathbf{r}(u, v) = (u \cos v, u \sin v, v). \]
   
   (a) Consider the vertical line segment \( L = \{ u = 0 \} \) in \( D \). Describe geometrically the image of \( L \) under \( \mathbf{r} \).
   
   (b) Repeat for the vertical segments where \( u = -1 \) and \( u = 1 \).
   
   (c) Use your answers in (a) and (b) to make a sketch of \( S \).

4. Consider the ellipsoid \( E \) given by \( \frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1 \).
   
   (a) Draw a picture of \( E \).
   
   (b) Find a parameterization of \( E \). Hint: Find a transformation \( T: \mathbb{R}^3 \to \mathbb{R}^3 \) which takes the unit sphere \( S \) to \( E \), and combine that with our existing parameterization of the plain sphere \( S \).