**Integrating vector fields.**

1. Consider the vector field \( \mathbf{F} = (y, 0) \) on \( \mathbb{R}^2 \).

   (a) Draw a sketch of \( \mathbf{F} \) on the region where \(-2 \leq x \leq 2 \) and \(-2 \leq y \leq 2 \). Check your answer with the instructor.

   (b) Consider the following two curves which **start** at \( A = (-2, 0) \) and **end** at \( B = (2, 0) \), namely the line segment \( C_1 \) and upper semicircle \( C_2 \).

   Add these curves to your sketch, and compute both \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \) and \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \). Check your answers with the instructor.

   (c) Based on your answer in (b), could \( \mathbf{F} \) be \( \nabla f \) for some \( f : \mathbb{R}^2 \to \mathbb{R} \)? Explain why or why not.

2. Consider the curve \( C \) and vector field \( \mathbf{F} \) shown below.

   ![Diagram of vector field and curve C]

   (a) Calculate \( \mathbf{F} \cdot \mathbf{T} \), where here \( \mathbf{T} \) is the unit tangent vector along \( C \). Without parameterizing \( C \), evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) by using the fact that it is equal to \( \int_C \mathbf{F} \cdot \mathbf{T} \, ds \).

   (b) Find a parameterization of \( C \) and a formula for \( \mathbf{F} \). Use them to check your answer in (a) by computing \( \int_C \mathbf{F} \cdot d\mathbf{r} \) explicitly.

3. Consider the points \( A = (0, 0) \) and \( B = (\pi, -2) \). Suppose an object of mass \( m \) moves from \( A \) to \( B \) and experiences the constant force \( \mathbf{F} = -mg \mathbf{j} \), where \( g \) is the gravitational constant.

   (a) If the object follows the straight line from \( A \) to \( B \), calculate the work \( W \) done by gravity using the formula from the first week of class.
(b) Now suppose the object follows half of an inverted cycloid $C$ as shown below. Explicitly parameterize $C$ and use that to calculate the work done via a line integral.

(c) Find a function $f : \mathbb{R}^2 \to \mathbb{R}$ so that $\nabla f = \mathbf{F}$. Use the Fundamental Theorem of Line Integrals to check your answers for (a) and (b). Have you seen the quantity $-f$ anywhere before? If so, what was its name?

4. If you get this far, work #52 from Section 16.2:

48. Experiments show that a steady current $I$ in a long wire produces a magnetic field $\mathbf{B}$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (as in the figure). Ampère’s Law relates the electric current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

where $I$ is the net current that passes through any surface bounded by a closed curve $C$, and $\mu_0$ is a constant called the permeability of free space. By taking $C$ to be a circle with radius $r$, show that the magnitude $B = |\mathbf{B}|$ of the magnetic field at a distance $r$ from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$