Problems:

1. Prove the following result that was used in class. Suppose $E$ is the elementary matrix obtained from $I_n$ by the row operation $R$, that is, $I_n \xrightarrow{R} E$. Prove that for all $A \in M_{n \times n}(\mathbb{R})$ one has $A \xrightarrow{R} EA$. Said another way, left-multiplication by $E$ implements the row operation that built $E$ in the first place.

2. Prove that if $A, B \in M_{n \times n}(\mathbb{R})$ are similar matrices then $\det(A) = \det(B)$.

3. A matrix $Q \in M_{n \times n}(\mathbb{R})$ is called orthogonal if $QQ^t = I_n$.
   
   (a) Prove that if $Q$ is orthogonal then $\det(Q) = \pm 1$.
   
   (b) Give examples of orthogonal matrices for $n = 2$ with both possible values of the determinant.

4. Suppose $A, B \in M_{n \times n}(\mathbb{R})$ satisfy $AB = I_n$.
   
   (a) Use the determinant to prove that $A$ is invertible.
   
   (b) Prove or disprove: $B = A^{-1}$.

5. Section 5.1 of [FIS], Problem 2 parts (a) and (c).

6. Let $T$ be a linear operator on a finite-dimensional vector space $V$.
   
   (a) Show that $T$ is invertible if and only if 0 is not an eigenvalue of $T$.
   
   (b) If $T$ is invertible, show that $\lambda^{-1}$ is an eigenvalue of $T^{-1}$ if and only if $\lambda$ is an eigenvalue of $T$.

7. Suppose $T : V \to V$ is a linear operator with $V$ finite-dimensional. Suppose $v \in V$ is an eigenvector of $T$ with eigenvalue $\lambda$. As usual, $T^m : V \to V$ denotes composition of $T$ with itself $m$ times. Prove that $v$ is also an eigenvector for $T^m$ and give a formula for the corresponding eigenvalue.

8. Section 5.1 of [FIS], Problem 3(a).

9. Section 5.1 of [FIS], Problem 4 parts (b) and (h).