
Webpage: http://dunfield.info/416


Textbooks: In the assignment, the two texts are abbreviated as follows:


Problems:

1. (a) Suppose $A$ is an $m \times n$ matrix with $m < n$. Show that the null space $\mathcal{N}(A)$ contains a nonzero vector by an argument involving the reduced row echelon form of $A$.

(b) Use part (a) to prove that any $j$ vectors in $\mathbb{R}^k$ are linearly dependent if $j > k$.

2. (a) Suppose $S$ is a subset of a vector space $V$. Show that if $v \in V$ is contained in $\text{span}(S)$, then $\text{span}(S) = \text{span}(S \cup \{v\})$.

(b) From problem 2(c) on the last HW, consider $V = \mathbb{R}^2$ and $S = \{(x, y) \mid x \geq 0 \text{ and } y \geq x \}$. Use part (a) to give a short proof that $\text{span}(S) = \mathbb{R}^2$ by showing that $\text{span}(S)$ contains the vectors $(1, 0)$ and $(0, 1)$.

3. Let $u$ and $v$ be distinct vectors in a vector space $V$. Show that $\{u, v\}$ is linearly dependent if and only if one of $u$ or $v$ is a scalar multiple of the other.

4. Either prove or give a counterexample to the following statement: If $u_1, u_2, u_3$ are three vectors in $\mathbb{R}^3$ none of which is a scalar multiple of another, then they are linearly independent.

5. In the vector space $F(\mathbb{R}, \mathbb{R})$ consider the elements $f(t) = \sin(t)$ and $g(t) = \cos(t)$. Is the subset $\{f, g\}$ linearly dependent or linearly independent? Prove your answer.

6. Section 1.6 of [FIS], Problem 1.

7. Section 1.6 of [FIS], Problem 2, parts (a) and (b).

8. Section 1.6 of [FIS], Problem 8.

9. Recall from HW 1 that the subset $U$ of all upper triangular matrices in $M_{n \times n}(\mathbb{R})$ forms a subspace. Find a basis for $U$ and use it to compute the dimension of $U$.

10. Suppose $W$ is a subspace of a finite-dimensional vector space $V$. For some $v \in V$ not in $W$, set $X = \text{span}(W \cup \{v\})$. Prove that $\dim(X) = \dim(W) + 1$. 