**Integrating vector fields.**

1. Consider the vector field \( \mathbf{F} = (y, 0) \) on \( \mathbb{R}^2 \).

   (a) Draw a sketch of \( \mathbf{F} \) on the region where \(-2 \leq x \leq 2 \) and \(-2 \leq y \leq 2 \). Check your answer with the instructor.

   (b) Consider the following two curves which start at \( A = (-2, 0) \) and end at \( B = (2, 0) \), namely the line segment \( C_1 \) and upper semicircle \( C_2 \).

   Add these curves to your sketch, and compute both \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \) and \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \). Check you answers with the instructor.

   (c) Based on your answer in (b), could \( \mathbf{F} \) be \( \nabla f \) for some \( f : \mathbb{R}^2 \to \mathbb{R} \)? Explain why or why not.

2. Consider the curve \( C \) and vector field \( \mathbf{F} \) shown below.

   (a) Calculate \( \mathbf{F} \cdot \mathbf{T} \), where here \( \mathbf{T} \) is the unit tangent vector along \( C \). Without parameterizing \( C \), evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) by using the fact that it is equal to \( \int_C \mathbf{F} \cdot \mathbf{T} \, ds \).

   (b) Find a parameterization of \( C \) and a formula for \( \mathbf{F} \). Use them to check your answer in (a) by computing \( \int_C \mathbf{F} \cdot d\mathbf{r} \) explicitly.

3. Consider the points \( A = (0, 0) \) and \( B = (\pi, -2) \). Suppose an object of mass \( m \) moves from \( A \) to \( B \) and experiences the constant force \( \mathbf{F} = -mg \mathbf{j} \), where \( g \) is the gravitational constant.

   (a) If the object follows the straight line from \( A \) to \( B \), calculate the work \( W \) done by gravity using the formula from the first week of class.
(b) Now suppose the object follows half of an inverted cycloid $C$ as shown below. Explicitly parameterize $C$ and use that to calculate the work done via a line integral.

![Diagram of an inverted cycloid](image)

(c) Find a function $f: \mathbb{R}^2 \to \mathbb{R}$ so that $\nabla f = F$. Use the Fundamental Theorem of Line Integrals to check your answers for (a) and (b). Have you seen the quantity $-f$ anywhere before? If so, what was its name?

4. If you get this far, work #52 from Section 16.2:

48. Experiments show that a steady current $I$ in a long wire produces a magnetic field $B$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (as in the figure). *Ampère’s Law* relates the electric current to its magnetic effects and states that

$$\int_C B \cdot dr = \mu_0 I$$

where $I$ is the net current that passes through any surface bounded by a closed curve $C$, and $\mu_0$ is a constant called the permeability of free space. By taking $C$ to be a circle with radius $r$, show that the magnitude $B = |B|$ of the magnetic field at a distance $r$ from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$