1. Let $R$ be the region shown which is bounded by the curve 
$y^2 - x - 2 = 0$, the line $y = x$, and $x$-axis. Evaluate $\iint_R 3y \, dA$.

(4 points)

\[
\int_0^2 \int_{y^2-2}^{\frac{y}{3}} 3y \, dx \, dy = \int_0^2 3y(y+2-y^2) \, dy
\]

\[
= \int_0^2 3y^2 + 6y - 3y^3 \, dy = y^3 + 3y^2 - \frac{3}{4} y^4 \bigg|_{y=0}^{y=2}
\]

\[
= 8 + 12 - 3 \cdot \frac{16}{4} = 8
\]

\[
\iint_R 3y \, dA = 8
\]

2. The integral $\iint_R 2x^2 + 2y^2 + y \, dA$ has the form $\int_{-\pi/2}^{\pi/2} \int_0^\cos \theta \, ??? \, dr \, d\theta$ when converted into polar coordinates.

(a) Mark the box below the picture of the region that represents $R$. (1 point)

(b) Fill in the missing integrand to convert this integral into polar coordinates. (2 points)

\[
\iint_R 2x^2 + 2y^2 + y \, dA = \int_{-\pi/2}^{\pi/2} \int_0^\cos \theta \left(2r^2 + r\sin \theta \right) \, r \, dr \, d\theta.
\]
3. Consider the region \( R \) in the positive octant bounded by the cone \( z = \sqrt{x^2 + y^2} \) and the planes \( z = 1 \), \( x = 0 \), and \( y = x \). In each column below, exactly one of the iterated integrals computes \( \iiint_R x \, dV \). Determine which are the correct answers and mark the boxes next to them. (4 points)

\[
\begin{array}{cccc}
\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec \phi} \rho^3 \sin^2 \phi \cos \theta \, d\rho \, d\phi \, d\theta & \checkmark & \int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{\sec \phi} \rho^3 \sin^2 \phi \cos \theta \, d\rho \, d\phi \, d\theta \\
\int_{\pi/4}^{\pi/2} \int_0^{\pi/4} \int_0^{\sec \phi} \rho^2 \sin^2 \phi \cos \theta \, d\rho \, d\phi \, d\theta & \checkmark & \int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{\sec \phi} \rho^2 \sin^2 \phi \cos \theta \, d\rho \, d\phi \, d\theta \\
\end{array}
\]

\[
\begin{array}{cccc}
\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^{z^2} r^2 \cos \theta \, dr \, d\theta \, dz & \checkmark & \int_0^1 \int_{\pi/4}^{\pi/2} \int_0^z r \cos \theta \, dr \, d\theta \, dz \\
\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^z r^2 \cos \theta \, dr \, d\theta \, dz & \checkmark & \int_0^1 \int_{\pi/4}^{\pi/2} \int_0^z r^2 \cos \theta \, dr \, d\theta \, dz \\
\end{array}
\]
4. A rectangular metallic plate \( R \) is placed in the plane with vertices at \((-2, -1), (-2, 1), (2, -1), \) and \((2, 1)\). The density \((\text{in } g/cm^2)\) of the plate, \(\rho(x, y)\), at various points is shown in the table, where \(x\) and \(y\) are measured in cm. Circle the best estimate for the mass of the plate. (2 points)

<table>
<thead>
<tr>
<th>(\rho(x, y))</th>
<th>(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>(-1/2)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

Mass of \( R \) = \[\begin{array}{ccccccccc}
0 & 4 & 15 & 30 & 46 & 60 & 78 & \text{grams.}
\end{array}\]

5. The integral of the function \( f(x, y, z) = 2x \) over a region \( R \) is computed by \[\int_0^1 \int_0^y \int_{y-x}^z 2x \, dz \, dx \, dy.\] Mark the box below the picture of the region \( R \). (2 points)
6. Suppose $R$ is the region in the first quadrant between the ellipses $x^2 + \frac{y^2}{4} = 1$ and $x^2 + \frac{y^2}{4} = 4$ and the lines $y = 0$ and $y = 2x$ shown at the right. Using the transformation

$$T(u, v) = \left( u \cos(v), 2u \sin(v) \right)$$

find the integrand and limits of integration expressing the integral $\iint_R x \, dA$ as an iterated integral over a subset $S$ in the $uv$-plane with $T(S) = R$. (5 points)

$$\int_1^2 \int_0^{\pi/4} (u \cos(v))(2u) \, dv \, du$$

$$\iiint_R x \, dA = \int_0^{\pi/4} \int_1^2 (u \cos(v)) 2u \, dudv.$$

Note: The order of integration is already determined.
7. Compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F} = \langle y + 2 \cos(x), 3x + e^{x^2} \rangle \) and \( C \) is the oriented curve shown.

(5 points)

\[
\begin{align*}
\int_C \mathbf{F} \cdot d\mathbf{r} &= \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA \\
&= \iint_D \left( \frac{\partial}{\partial x} (3x + e^{x^2}) - \frac{\partial}{\partial y} (y + 2 \cos(x)) \right) \, dA \\
&= \iint_D (3 - 1) \, dA = 2 \text{Area}(D) \\
&= 2 \left( \frac{7}{2} \right) = 7
\end{align*}
\]

\[ \int_C \mathbf{F} \cdot d\mathbf{r} = 7 \]

8. Let \( R \) be the rectangle whose vertices are \((1, 0), (2, 1), (3, -2)\), and \((4, -1)\) shown at the right.

(a) Exactly one of the following defines a transformation \( T(u, v) \) from the \( uv\)-plane to the \( xy\)-plane with \( T(S) = R \), where \( S = \{(u, v) \mid 0 \leq u \leq 1, 0 \leq v \leq 1\} \). Circle the correct formula for \( T(u, v) \).

\[
\begin{align*}
\langle 2u + 3v, u - 2v \rangle & \quad \langle 2u + 4v, u - v \rangle & \quad \langle 2u + 4v + 1, u - v \rangle \\
\langle 2u + 3v + 1, u - 2v \rangle & \quad \langle u + 2v, u - 2v \rangle & \quad \langle u + 2v + 1, u - 2v \rangle
\end{align*}
\]

(b) \( \iint_R y \, dA \) is negative, zero, positive

(1 point)

scratch space

\[
A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}
\]

\[
(u, v) \mapsto A \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} u + 2v + 1 \\ u - 2v \end{pmatrix}
\]
9. Consider the surface \( S \) parameterized by \( \mathbf{r}(u, v) = \langle u^2 \sin v, u, u^2 \cos v \rangle \) for \( 0 \leq u \leq 1 \) and \( 0 \leq v \leq 2\pi \).

(a) Mark the box below the best picture of \( S \). (1 point)

(b) Circle the correct formula for \( \mathbf{r}_u \times \mathbf{r}_v \). (2 points)

\[
\langle u^2 \sin v, u^3, u^2 \cos v \rangle \quad \langle \cos v, u^2, u \sin v \rangle \quad \langle -u^2 \sin v, 2u^3, -u^2 \cos v \rangle \quad \langle -u \cos v, 2u^2, -u \sin v \rangle
\]

(c) Circle the integrand for the integral \( \int_0^1 \int_0^{2\pi} g(u, v) \, dv \, du \) that computes the surface area of \( S \). (2 points)

\[
g(u, v) = \sqrt{u^4 + u^6} \quad \sqrt{u^2 + u^4} \quad \sqrt{4u^4 + 4u^6} \quad \sqrt{4u^2 + 4u^4} \quad \sqrt{u^4 + 4u^6} \quad \sqrt{u^2 + 4u^4}
\]

(d) \( \iint_S xz \, dS \) is \( \text{negative} \) \( \text{zero} \) \( \text{positive} \) (1 point)

---

**Scratch Space**

\[
\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2u \sin v & 1 & 2u \cos v \\
u^2 \cos v & 0 & -u^2 \sin v \\
\end{vmatrix} = \langle -u^2 \sin v, 2u^3, -u^2 \cos v \rangle
\]

\[
|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{u^4 \sin^2 v + 4u^6 + u^4 \cos^2 v} = \sqrt{u^4 + 4u^6}
\]
10. Parameterize each of the surfaces below with a function \( r(u, v) \). Be sure to specify the domain \( D \) of your parameterization.

(a) The portion of the sphere \( x^2 + y^2 + z^2 = 4 \) where \( y \geq 0 \) and \( z \geq 0 \). (3 points)

\[
r(u,v) = \begin{pmatrix} 2 \sin u \cos v, 2 \sin u \sin v, 2 \cos u \end{pmatrix}
\]

\[
D = \left\{ (u,v) \mid 0 \leq u \leq \frac{\pi}{2}, \quad 0 \leq v \leq \pi \right\}
\]

(b) The part of the graph \( x = 1 - z^2 \) where \( x \geq 0 \) and \( -2 \leq y \leq 2 \). (4 points)

\[
r(u,v) = \begin{pmatrix} 1 - \sqrt{2}, \quad u, \quad v \end{pmatrix}
\]

\[
D = \left\{ (u,v) \mid -2 \leq u \leq 2, \quad -1 \leq v \leq 1 \right\}
\]