1. Consider the vector field \( F(x, y) = \langle y + e^x, x - \cos y \rangle \). Find a function \( f(x, y) \) such that \( F = \nabla f \). \( \text{(2 points)} \)

\[ f(x, y) = \]

2. For each of the given regions \( D \) in \( \mathbb{R}^2 \) below, circle the phrase that makes the sentence true. \( \text{(1 point each)} \)

(a) A continuous function on \( D = \{ x^2 + 4y^2 \geq 5 \} \)

must might or might not must not have an absolute maximum.

(b) A continuous function on \( D = \{ x^2 + 4y^2 \leq 5 \} \)

must might or might not must not have an absolute maximum.

(c) A continuous function on \( D = \{ x^2 + 4y^2 < 5 \} \)

must might or might not must not have an absolute maximum.

(d) A continuous function on \( D = \{ x^2 + 4y^2 = 5 \} \)

must might or might not must not have an absolute maximum.

3. Suppose \( f(x, y) \) is a differentiable function with continuous second order partial derivatives and values given by the table below.

\[
\begin{array}{c|cc|cccc}
(x, y) & f(x, y) & f_x(x, y) & f_y(x, y) & f_{xx}(x, y) & f_{yy}(x, y) & f_{xy}(x, y) \\
\hline
(-1, 0) & 4 & 0 & 0 & -2 & -3 & 2 \\
(0, 1) & 0 & 0 & 1 & 1 & 2 & 0 \\
(2, 1) & -2 & 0 & 0 & 1 & 1 & 3 \\
\end{array}
\]

For each of the given points, circle the best description of the point. \( \text{(1 point each)} \)

(-1, 0) not critical local minimum local maximum saddle point undetermined

(0, 1) not critical local minimum local maximum saddle point undetermined

(2, 1) not critical local minimum local maximum saddle point undetermined
4. Find the maximum and minimum values of the function \( f(x, y) = -x + 2y \) on the curve \( x^2 + 2y^2 = 3 \). (5 points)

Minimum value = 

Maximum value = 
5. The contour map of a differentiable function \( g \) is shown at right. For each part, circle the best answer. (2 points each)

(a) The directional derivative \( D_v g(P) \) is:
   
   positive  negative  zero

(b) The vector \( u \) is parallel to \( \nabla g(Q) \).
   
   True  False

(c) Estimate \( \int_C g(x, y) \, ds \):
   
   \[ -12 \quad -9 \quad -6 \quad -3 \quad 0 \quad 3 \quad 6 \quad 9 \quad 12 \]

(d) Find \( \int_C \nabla g \cdot dr \):
   
   \[ -12 \quad -9 \quad -6 \quad -3 \quad 0 \quad 3 \quad 6 \quad 9 \quad 12 \]

6. Consider the curve \( C \) in \( \mathbb{R}^3 \) whose projections onto the \( xy \), \( xz \) and \( yz \) planes are:

Check the box below the three-dimensional plot of \( C \). (2 points)
7. Let \( C \) be the curve in three-dimensional space parametrized by \( \mathbf{r}(t) = (2 \cos t, 2 \sin t, t) \) for \(-\pi \leq t \leq \pi\).

(a) Find the mass of a thin wire in the shape of \( C \), if the density function is \( \rho(x, y, z) = x + z + 10 \). \( \text{(5 points)} \)

\[
\text{Mass} =
\]

(b) Suppose that a particle moves along \( C \) starting at \( \mathbf{r}(-\pi) = (-2, 0, -\pi) \) and ending at \( \mathbf{r}(\pi) = (-2, 0, \pi) \). Find the work done on the particle by the force \( \mathbf{F}(x, y, z) = yi - xj \). \( \text{(5 points)} \)

\[
\text{Work} =
\]
8. (a) Let $S_1$ be the surface defined by $x = 2z^2 + y^2$ (an elliptic paraboloid). Find an equation for the tangent plane to $S_1$ at the point $(3, -1, 1)$. (4 points)

Equation: $x + y + z = \underline{\phantom{0}}$

(b) Let $S_1$ be as in the previous part, and let $S_2$ be the surface defined by $y^2 + \frac{z^2}{4} = 1$ (a cylinder over an ellipse). Find a vector function $\mathbf{r}(t)$ that parametrizes the curve that is the intersection of the surfaces $S_1$ and $S_2$. Specify the range of the parameter values so that the function traces the curve exactly once. (4 points)

$\mathbf{r}(t) = \langle \underline{\phantom{0}}, \underline{\phantom{0}}, \underline{\phantom{0}} \rangle$ for $\underline{\phantom{0}} \leq t \leq \underline{\phantom{0}}$
9. Check the box below the picture of the curve \( \mathbf{r}(t) = (\sin t, \cos^2 t) \), \( 0 \leq t \leq 2\pi \). \hspace{1cm} (2 points)

10. A vector field \( \mathbf{G} \) is plotted at right.

   (a) Circle the formula for \( \mathbf{G} \). \hspace{1cm} (1 point)
   \[ x\mathbf{i} + y\mathbf{j} - x\mathbf{i} - y\mathbf{j} \]

   (b) \( \mathbf{G} \) is conservative. \hspace{1cm} (1 point)
   \[ \text{True} \quad \text{False} \]

11. The region \( D \) defined by \( \{0.03 < x^2 + y^2 < 1.3\} \) is shown at right. Within this region are three curves \( A, B, C \). Each curve starts at \((0, -1)\) and ends at \((0, 1)\). Suppose that \( \mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j} \) is a differentiable vector field defined on \( D \) with the properties

   \[ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \int_A \mathbf{F} \cdot d\mathbf{r} = -1, \quad \text{and} \quad \int_C \mathbf{F} \cdot d\mathbf{r} = 2. \]

   (a) The region \( D \) is simply connected. \hspace{1cm} (1 point)
   \[ \text{True} \quad \text{False} \]

   (b) \( \mathbf{F} \) is conservative. \hspace{1cm} (1 point)
   \[ \text{Yes} \quad \text{No} \quad \text{Cannot determine} \]

   (c) Find \( \int_B \mathbf{F} \cdot d\mathbf{r} \). \hspace{1cm} (1 point)
   \[ -3 \quad -2.5 \quad -2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \]