1. (a) Sketch the first-octant portion of the sphere $x^2 + y^2 + z^2 = 5$. Check that $P = (1, 1, \sqrt{3})$ is on this sphere and add this point to your picture.

(b) Find a function $f(x, y)$ whose graph is the top-half of the sphere.

(c) Imagine an ant walking along the surface of the sphere. It walks down the sphere along the path $C$ that passes through the point $P$ in the direction parallel to the $yz$-plane. Draw this path in your picture.

(d) Use the function from (b) to find a parameterization $\mathbf{r}(t)$ of the ant's path along the portion of the sphere shown in your picture. Specify the domain for $\mathbf{r}$, i.e. the initial time when the ant is at $P$ and the final time when it hits the $xy$-plane.

2. Consider the curve $C$ in $\mathbb{R}^3$ given by

$$\mathbf{r}(t) = (e^t \cos t) \mathbf{i} + 2\mathbf{j} + (e^t \sin t) \mathbf{k}$$

(a) Calculate the length of the segment of $C$ between $\mathbf{r}(0)$ and $\mathbf{r}(t_0)$. Check your answer with the instructor.

(b) Suppose $h: \mathbb{R} \to \mathbb{R}$ is a function. We can get another parameterization of $C$ by considering the composition

$$\mathbf{f}(s) = \mathbf{r}(h(s))$$

This is called a reparameterization. Find a choice of $h$ so that

i. $\mathbf{f}(0) = \mathbf{r}(0)$

ii. The length of the segment of $C$ between $\mathbf{f}(0)$ and $\mathbf{f}(s)$ is $s$. (This is called parameterizing by arc length.)

Check your answer with the instructor.

(c) Without calculating anything, what is $|\mathbf{f}'(s)|$?

(d) Draw a sketch of $C$.

3. Consider the curve $C$ given by the parameterization $\mathbf{r}: \mathbb{R} \to \mathbb{R}^3$ where $\mathbf{r}(t) = (\sin t, \cos t, \sin^2 t)$.

(a) Show that $C$ is in the intersection of the surfaces $z = x^2$ and $x^2 + y^2 = 1$.

(b) Use (a) to help you sketch the curve $C$.

4. As in 2(b), consider a reparameterization

$$\mathbf{f}(s) = \mathbf{r}(h(s))$$

of an arbitrary vector-valued function $\mathbf{r}: \mathbb{R} \to \mathbb{R}^3$. Use the chain rule to calculate $|\mathbf{f}'(s)|$ in terms of $\mathbf{r}'$ and $h'$. 

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**Curves and integration.**

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Thursday, October 6